Combined Precoding and Volterra Equalization for the Mitigation of Fiber-Optic Nonlinear Channel Impairments

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Abstract

We investigate the performance of a combined Tomlinson-Harashima precoding (THP) and a Volterra nonlinear equalization (VNLE) in a nonlinear fiber-optic communication system aiming at a data rate of 100 Gbits/sec. THP-VNLE is compared with a feed-forward VNLE and a combined VNLE decision-feedback equalizer (DFE) by means of performance and computational complexity. Two different transmission scenarios, both using a 25 GBaud 16QAM signal, are introduced, and the performance of the different equalizers is assessed qualitively, and via Monte-Carlo simulations for the measurement of the BER.

1 Introduction

Assuming transmission with a sufficiently low launch-power, a fiber-optic transmission link can be characterized as a linear and dispersive channel. The inter-symbol interference (ISI) associated with the channel is the result of two effects: The first is the fiber's chromatic dispersion (CD); the second is the effect of filtering, which can be originated from transmit and receive filters, band-pass filters, and bandwidth-limitation of electrical and optical devices, e.g. digital-to-analog converters (DACs), analog-to-digital converters (ADCs), and reconfigurable optical add-drop multiplexers (ROADMs).

Using coherent detection at the receiver side, the resulting ISI can be efficiently compensated using digital signal processing (DSP) techniques. Equalization can be done either with a plain feed-forward equalizer (FFE) or a combined FFE and a decision-feedback equalizer (DFE). In addition, in order to avoid its error propagation effect, and to allow a straight-forward application with channel-coding schemes, the feedback structure of the DFE can be replaced by a precoder at the transmitter side [1]: a structure referred to in the literature as *Tomlinson-Harashima precoding* (THP) [2,3]. Considering the joint equalization of CD and filtering-induced ISI, THP was shown to outperform FFE and to offer a similar performance as of DFE, with the advantage of the elimination of error propagation, which was shown to be potentially catastrophic in the vicinity of the FEC limit [4,5].

Due to the increasing demand for high data rates and long reach, transmission with higher launch-power becomes inevitable, e.g. in order to meet the required optical signal to noise ratio (OSNR). Unfortunately, by increasing the launch-power, transmission will suffer not only from ISI, but fiber nonlinearities as well, namely self-phase modulation (SPM) for single-carrier transmission. Since fiber nonlinearities cannot be compensated using an equalizer designed for linear ISI, the equalizers mentioned above have to be redesigned in order to be applied in the nonlinear regime. This can be achieved using a Volterra nonlinear equalizer (VNLE) [6,7], which can replace the FFE structure, or can be combined with the DFE/THP feedback structure.

In this paper, we investigate the performance of the combined THP-VNLE in a nonlinear fiber-optic communication system. The remainder of the paper is organized as follows: Section 2 summarizes the theory and realization of a VNLE. The simulation results and their analyses are given in section 3. Finally, section 4 concludes this paper.

2 Volterra Equalization for Compensation of Nonlinearities

2.1.1 Theoretical Background

The Volterra theory is a generalization of linear time invariant (LTI) systems to the nonlinear case. It states that every nonlinear, time-invariant (NLTI) system can be fully defined by an infinite sum of multidimensional convolution integrals of increasing order, as written in the following equation:

$$\begin{aligned} y_{NL}(t) = & \int_{-\infty}^{\infty} h_0(\tau_1)x(t-\tau_1)\,\mathrm{d}\tau_1 + \\ & + \int_{-\infty}^{\infty}\!\!\int h_1(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2 + \\ & + \int_{-\infty}^{\infty}\!\!\dots\int_{-\infty}^{\infty} h_n(\tau_1,\dots,\tau_n)x(t-\tau_1)\dots x(t-\tau_n)\,\mathrm{d}\tau_1\dots\mathrm{d}\tau_n. \end{aligned} \quad (1.1)$$

Here, h_n is the n-th order Volterra kernel, where h_0 represents the linear part of the system and the Volterra kernels of higher order represent its nonlinear behavior. Two conclusions can be drawn from the Volterra theory: First, the knowledge of all Volterra kernels of a system is sufficient in order to obtain its response to any arbitrary input signal [8]. Second, when the system investigated is in fact composed from an infinite sum of convolution integrals, an accurate model of the system is not realizable, since it has to be truncated into a finite sum. Unfortunately, the last conclusion holds for an optical fiber. In order to illustrate the existence of an infinite number of Volterra kernels, we examine the closed-form solution of the nonlinear Schrödinger equation (NLSE) for a nonlinear fiber of length L, omitting the effect of dispersion [9]:

$$A(z=L,t)=e^{-\frac{\alpha}{2}L}\cdot e^{-j\gamma|A(0,t)|^2 L_{\mathrm{eff}}}A(0,t)\,, \qquad (1.2)$$

where α, γ, and L_{eff} are the attenuation coefficient, nonlinear coefficient and the effective length of the fiber, respectively. It is possible to expand the second exponential with Taylor series to the following infinite sum:

$$A(L,t)=e^{-\frac{\alpha}{2}L}\left(1+\gamma|A(0,t)|^2+\frac{1}{2!}\gamma^2|A(0,t)|^4+\right. \\ \left.+\frac{1}{3!}\gamma^3|A(0,t)|^6+\ldots\right)A(0,t). \qquad (1.3)$$

As can be seen, the output signal $A(L,t)$ is composed of a linear term of the input signal $A(0,t)$ and an infinite sum of higher order *odd* powers of the input (for instance $|A(0,t)|^4\cdot A(0,t)=|A(0,t)|^5\,e^{j\arg\{A(0,t)\}}$). When the effect of dispersion is included, the fiber is still a nonlinear system with an infinite sum of nonlinear terms. Note that the Taylor coefficients decrease considerably with increasing order. Therefore, depending on the fiber launch power, the high order nonlinear terms can be neglected. This is done in practice in order to model the nonlinear fiber [10], or, as will be discussed in the following section, to realize an equalizer for the joint compensation of linear and nonlinear impairments.

2.1.2 Realization

A Volterra nonlinear equalizer (VNLE) should in general provide us with a model of the inverse of the transmission system. From a practical point-of-view, the VNLE has to be of a finite-order. The realization can be done either in the frequency domain [10], or entirely in the time domain, as will be shown here.

Finding the VNLE coefficients is in fact an estimation problem. Using the minimum mean-squared error (MMSE) criterion, this problem can be considered as a generalization of Wiener filtering [11]. Thanks to the linear relation between the equalizer coefficients and its output, the coefficients can be computed in the same way as for a linear Wiener filter, e.g. by solving the Wiener-Hopf equations. The quality of estimation depends heavily on our choice of the system model. It is clear that when applying a linear model to estimate a nonlinear system, the Wiener filter will be suboptimal. However, also the nonlinear model of the system should be chosen carefully, to assure the best performance. By observing (1.1) and (1.3), and restricting to third order nonlinearities, the model for the equalizer can be given as [6]:

$$y_k=\sum_{\kappa=0}^{N_e-1}e_\kappa x_{k-\kappa}+\sum_{l=0}^{N_e-1}\sum_{m=l}^{N_e-1}\sum_{n=0}^{N_e-1}e_{lmn}x_{k-l}\cdot x_{k-m}\cdot x_{k-n}^*\,, \qquad (1.4)$$

where y_k and x_k are the output and input of the equalizer at time index k, e_κ and e_{lmn} are the equalizer's coefficients, which are a discrete representation of the first and third Volterra kernels of the equalizer, respectively, and N_e is the number of linear equalizer coefficients. Note that the term $x_{k-l}\cdot x_{k-m}\cdot x_{k-n}^*$ results from the term $|A(0,t)|^2\cdot A(0,t)=A(0,t)\cdot A(0,t)\cdot A^*(0,t)$ in (1.3). Observing (1.4), the number of nonlinear equalizer coefficients sums up to $0.5N_e^2(N_e+1)$. Now, the transmission system considered here is highly dispersive, and in the absence of in-line dispersion compensation (since we would like to compensate entirely using DSP techniques), it is characterized by a strong ISI. This results in a relatively large value for N_e, which correspondingly will drastically increase the complexity of the VNLE. Therefore, a static linear equalizer has to be applied in the first stage, i.e. before the nonlinear compensation, in order to compensate for as much dispersion as possible, shortening the length of the channel impulse response to a reasonable value. If the system to be equalized is considered as pseudo-linear [9], it is possible to reduce further the number of nonlinear equalizer coefficients by taking into account only the intra-channel nonlinear contributions with the strongest influence over neighbor symbols, which can be found in the same manner as in phase-matching in WDM systems [6,9].

3 Simulation Results

3.1 System Setup

A schematic of the system used for the investigation is given in **Figure 1**. A 2^{16} long De-Bruijn pseudo random hexadecimal sequence was mapped to 16QAM symbols d_k. A symbol rate of 25 GBaud was chosen, in order to achieve a gross data rate of 100 Gbits/sec. The data sequence was either processed with THP before the DAC, or directly converted to an analog signal in case of a system scenario with FFE or FFE-DFE. The DAC was modeled as a rectangular pulse shaping filter followed by a 5^{th} order Bessel low-pass filter with a cut-off frequency of 13.5 GHz to simulate its bandwidth limitations. Using a laser operating at a wavelength $\lambda_c = 1550$ nm and an optical IQ modulator, the analog signal was up-converted into the optical band-pass domain. The linewidth of the laser was set to zero in order to focus on the effect of SPM. The Mach-Zehnder modulators (MZMs) in the IQ modulator were assumed to operate in their linear range.

The optical signal was transmitted through a transmission link of either a single span or five spans. Each span was composed of a standard single-mode fiber (SSMF) with

an attenuation coefficient $\alpha_{\mathrm{dB}} = 0.2$ dB/km, a dispersion parameter $D = 17$ ps/nm/km, and a nonlinear coefficient $\gamma = 0.0013\ (\mathrm{W}\cdot\mathrm{km})^{-1}$; an EDFA with a noise figure (FN) of 5 dB for a complete compensation of the span loss, and a 2nd order super Gaussian optical band-pass filter with a cut-off frequency of 25 GHz to simulate a ROADM.
The received signal was detected by a homodyne coherent receiver, followed by a 5th order Butterworth anti-aliasing filter with a cut-off frequency of 17.5 GHz. The signal was then sampled with an ADC operating at 50 Gsamples/sec. As was stated before, the effect of LPN was neglected, so the system suffers mainly from SPM and SPM-induced phase noise.

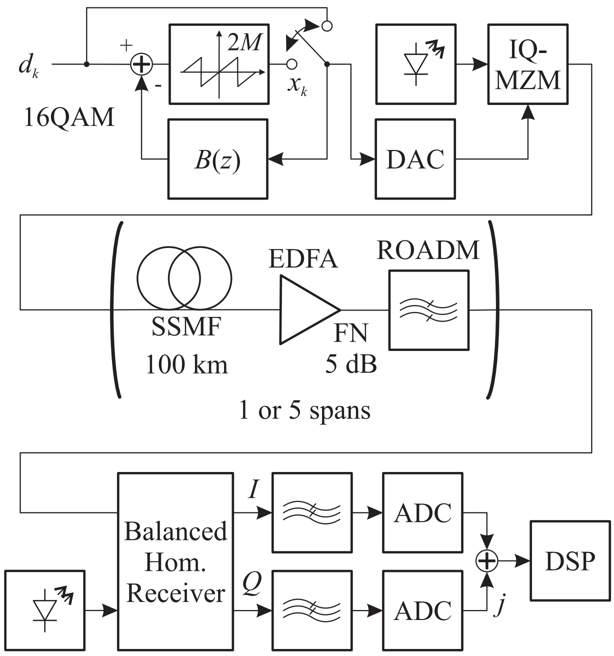

Figure 1 System setup

3.2 DSP Scheme

A block diagram of the investigated DSP scheme is shown in **Figure 2**. The received and sampled signal was first processed with a frequency-domain blind equalizer for CD compensation [12] using two samples per symbol, and then was down-sampled to the symbol rate. The signal was then processed with a plain FFE, a combined FFE-DFE, or an FFE, which works in conjunction with the THP. When considering VNLE in the results, all linear FFEs where replaced with *nonlinear* FFEs to allow equalization of fiber nonlinear effects. After equalization, the signal was detected, and performance was assessed with either the error vector magnitude (EVM) figure of merit or BER, aiming a resolution of 1000 bit errors. For that purpose, Monte-Carlo simulations were carried out with a maximum number of 1000 blocks, yielding ca. 4 million symbols or 16 million bits.

In the following sections, we denote the equalizer by its components, where each is followed by the number of *linear* coefficients used for that design. For instance: An FFE-DFE with 8 feed-forward coefficients and 4 feedback coefficients is denoted as FFE(8)-DFE(4).

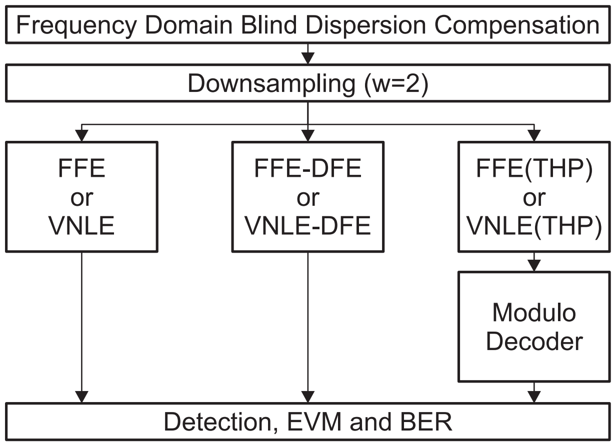

Figure 2 DSP scheme

3.3 Visualization of the Equalized Signals for 100 km Nonlinear Transmission

In the following scenario, the signal was transmitted through a single span at a launch power of 8 dBm. We assume a 10% dispersion estimation error for the blind dispersion compensation block. The number of coefficients used for each second stage equalizer is summarized in **Table 1**. The equalized constellations are shown in **Figure 3** and **Figure 4**.

	FFE(3)	FFE(4)	FFE(3)-DFE(1)	THP(1)-FFE(3)
N_e	3	4	3	3
N_b	0	0	1	1
N_{e3}	18	40	18	18
N_{tot}	21	44	22	22

Table 1 Number of equalizer coefficients for different equalizer designs. N_e: feed-forward; N_b: feedback; N_{e3}:VNLE; N_{tot}: total number of coefficients.

By examining Figure 3a., c. and e. for compensation of ISI only, the results show clearly the nonlinear behavior of the system, as nonlinear phase distortions linger after the (partial) removal of the ISI. Notice that the FFE-DFE structure shows a tighter cluster composition than a plain FFE with the same number of coefficients. Nevertheless, it is possible to see symbols, which do not belong to any of the 16 clusters. These errors can be attributed to the DFE's error propagation effect.
In Figure 3b., d. and f. the positive effect of the VNLE on the received constellation is visible, and especially for the last two equalizer designs, the performance is greatly improved.

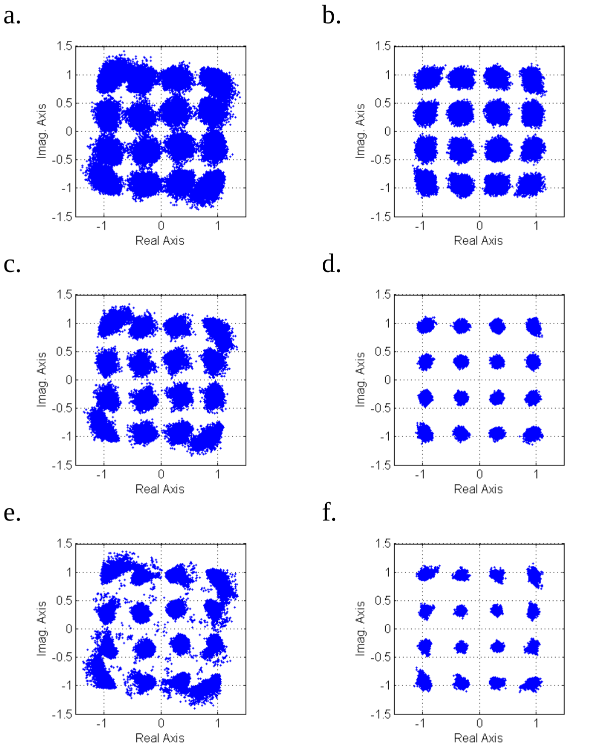

Figure 3 Equalized constellation diagrams: a. FFE(3), b. VNLE(3), c. FFE(4), d. VNLE(4), e. FFE(3)-DFE(1), f. VNLE(3)-DFE(1).

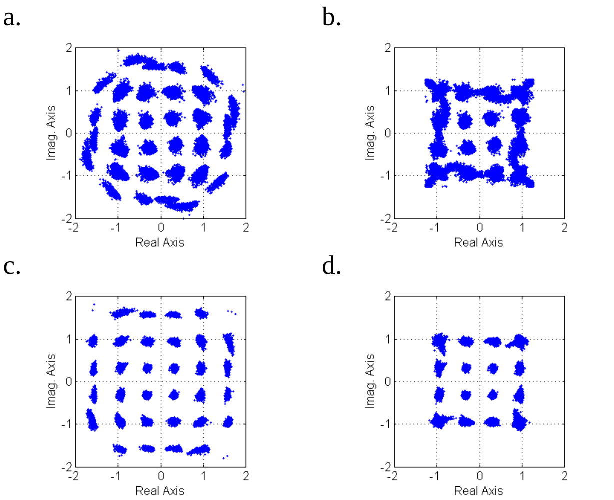

Figure 4 Equalized constellation diagrams for THP: a. THP(1)-FFE(3) before modulo decoding, b. THP(1)-FFE(3) after decoding, c. THP(1)-VNLE(3) before modulo decoding, d. THP(1)-VNLE(3) after decoding.

In the results for the design with THP, we first examine the so-called *effective data sequence* (EDS) [1] constellation shown in Figure 4a., i.e. the signal constellation *before* modulo decoding. In comparison with Figure 3e., the error propagation effect is no longer visible, as expected. However, this constellation exhibits larger values than the original constellation. These additional values are more susceptible to the SPM effect, since they are of greater instantaneous power. Without an additional phase correction for the outer ring, the modulo operator will falsely decode these values, resulting in the constellation shown in Figure 4b.

Combining now THP with VNLE, the system performance considerably improves as for the other designs. However, the fact that the signal after THP-VNLE features higher power values, system performance is slightly degraded compared with VNLE-DFE.

Last, the measured EVM values for the constellations mentioned above are summarized in **Table 2**. Note that we use the EVM here not as an estimation of the BER (such an estimation would be anyhow inaccurate, since nonlinear impairments are evident in the received constellation), but rather as a measurement for the quality of equalization. As can be seen from the table, when VNLE is combined with a feedback compensator (DFE or THP), it is possible to achieve better performance than a feed-forward VNLE, and at the same time the total number of coefficients can be reduced.

	FFE(3)	FFE(4)	FFE(3)-DFE(1)	THP(1)-FFE(3)
EVM_{lin} %	16.92	13.57	13.26	24.87
EVM_{nl} %	12.26	5.87	4.54	5.07
N_{tot}	21	44	22	22

Table 2 Measured EVM values for the different equalizer designs. EVM_{lin}: values for a design without VNLE. EVM_{nl}: values for a design with VNLE.

3.4 Monte-Carlo Simulations for 500 km Nonlinear Transmission

In the following scenario, the signal was transmitted through 5 spans at a varying launch power of –10 to 10 dBm, which corresponds to an OSNR of 16 to 36 dB for a *linear* fiber system. Again, we assume a 10% dispersion estimation error for the blind dispersion compensation block. We assume in this scenario a design constraint of 12 linear equalizer coefficients (the number of Volterra coefficient is not included), where in case of FFE-DFE and THP-FFE 8 coefficients are used for the feed-forward part, and 4 coefficients for the feedback part. The BER performance for each equalizer is shown vs. the fiber launch power in **Figure 5**.

Figure 5a. shows the nonlinear tolerance of the equalizers without VNLE. In this scenario, the FFE design with 12 coefficients merely manages to cross the FEC limit at a launch power of –2 dBm. However, using the same total number of coefficients, both FFE-DFE and THP-FFE completely outperform the FFE. The FFE-DFE shows in addition better performance than THP-FFE at -4 and -2 dBm, which is in accordance with the previous results.

In Figure 5b. we observe the BER performance of all equalizers, this time in conjunction with VNLE. Surprisingly, VNLE shows worse performance than its linear version. In addition, the performance of THP-VNLE seems to degrade for launch power lower than -4 dBm, but shows an increased nonlinear tolerance of 1.3 dB for launch power values greater than -2 dBm. VNLE-DFE is the only equalizer that shows an improvement throughout the whole launch power range. We conclude that the amount of residual dispersion at the input of the VNLE can severely impact its performance, and should be reduced as much as possible.

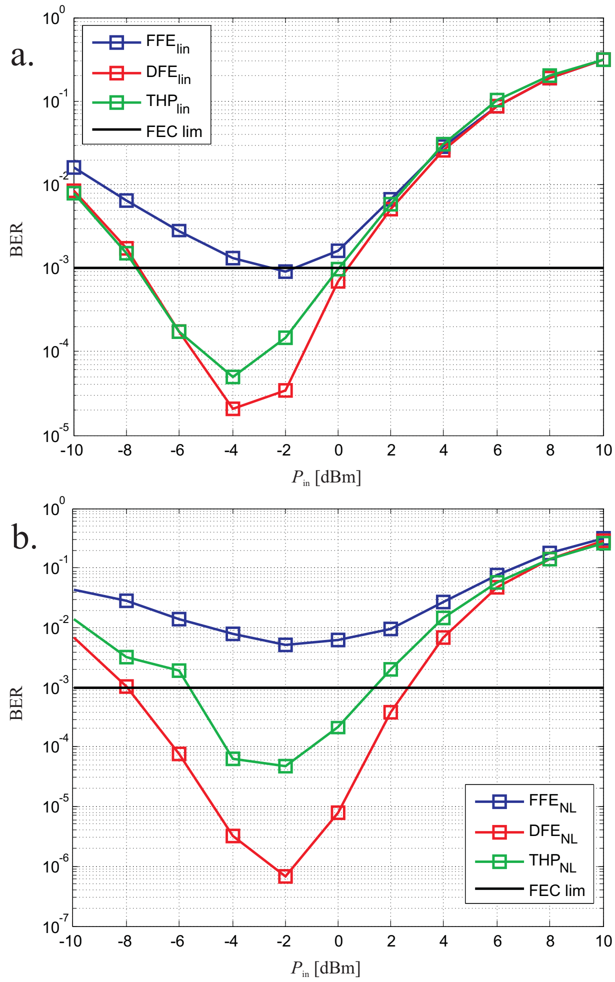

Figure 5 BER vs. launch power for 500 km transmission in case of a. linear equalization and b. VNLE.

4 Conclusion

In this paper, the performance of a combined THP-VNLE in a nonlinear fiber-optic communication system was investigated. THP was shown to outperform FFE, but was slightly outperformed by DFE for a short transmission distance of 100 km, due to the increased size of the effective data sequence in comparison with the original constellation. By applying VNLE equalization in conjunction with the investigated equalizers, system performance was in general improved. However, the effect of too high residual ISI in combination with SPM was shown to degrade the performance of THP-VNLE and plain VNLE, whereas VNLE-DFE remained relatively more robust to this effect.

5 References

[1] Robert F.H. Fischer, *Precoding and Signal Shaping for Digital Transmission*. New York: John Wiley & Sons, Inc, 2002.

[2] M. Tomlinson, "New Automatic Equaliser Employing Modulo Arithmetic," *Electronic Letters*, vol. 7, no. 5, pp. 138-139, March 1971.

[3] H. Harashima and H. Miyakawa, "Matched-Transmission Technique for Channels with Intersymbol Interference," *IEEE Transactions on Communications*, vol. 20, no. 4, pp. 774-780, August 1972.

[4] Roi Rath, Christian Schmidt, and Werner Rosenkranz, "Is Tomlinson-Harashima Precoding Suitable for Fiber-Optic Communication Systems?," in *14. ITG Fachtagung Photonische Netze*, Leipzig, 2013, pp. 61-67.

[5] Roi Rath and Werner Rosenkranz, "Tomlinson-Harashima Precoding for Fiber-Optic Communication Systems," in *ECOC*, London, 2013.

[6] Yan Gao, Fan Zhang, Liang Dou, Zhangyuan Chen, and Anshi Xu, "Intra-Channel Nonlinearities Mitigation in Pseudo-Linear Coherent QPSK Transmission Systems via Nonlinear Electrical Equalizer," *Optics Communications*, vol. 282, no. 12, pp. 2421 - 2425, 2009.

[7] Werner Rosenkranz and Johannes von Hoyningen-Huene, "Nonlinearity Compensation and Equalization in Access Networks," in *Proc. Optoelectronics and Communications Conference OECC*, Busan, South Korea, 2012.

[8] Martin Schetzen, *The Volterra and Wiener theories of nonlinear systems*, 2nd ed. New York: Wiley, 1980.

[9] Govind P. Agrawal, *Nonlinear Fiber Optics*, 3rd ed. London: Academic Press, 2001.

[10] Kumar V. Peddanarappagari and Maite Brandt-Pearce, "Volterra Series Transfer Function," *IEEE Journal of Lightwave Technology*, vol. 15, no. 12, pp. 2232-2241, December 1997.

[11] Simon Haykin, *Adaptive Filter Theory*, 3rd ed., Tom Robbins, Ed. New Jersey, United States of America: Prentice Hall, 1996.

[12] Riichi Kudo et al., "Coherent Optical Single Carrier Transmission Using Overlap Frequency Domain Equalization for Long-Haul Optical Systems," *Journal of Lightwave Technology*, vol. 27, no. 16, pp. 3721-3728, 2009.