Costas-Loop Based Carrier Recovery in Optical Coherent Intersatellite Communications Systems

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Abstract—Optical intersatellite communication is an attractive alternative to current RF satellite communication in order to increase the data rate and lower the power consumption. By using coherent detection, full signal recovery is achieved at the receiver which allows modulating both, amplitude and phase. However, the needed local oscillator requires a complex carrier recovery system. In order to achieve homodyne detection an optical phase-locked loop is used which is characterised by a nonlinear behaviour that influences the data transmission performance. We present a nonlinear characteristic investigation of this Costas-loop based carrier recovery system for optical high-speed intersatellite data transmission. We combine the fundamentals of future optical intersatellite communication with analyses of an optical phase-locked loop based carrier control system. The corresponding nonlinear differential equation system is derived and finally investigated by phase plane diagrams. The impact of noise as well as the cycle slip phenomena will be discussed.

Keywords—Costas-loop; optical phase-locked loop; coherent detection; carrier recovery; optical intersatellite link;

I. INTRODUCTION

The increasing number of real-time earth observation applications, e.g. faster earthquake or tsunami forecasts, requires high-speed communication links between satellites and between satellites and earth. Due to the lower power consumption and higher data rates, optical intersatellite links (OISL) offer an attractive alternative to conventional RF-communication. Furthermore, the narrow laser beam width ensures a better data security and the lower weight improves the cost efficiency [1]. The complex alignment of both satellites to achieve line-of-sight (LOS) connection is realised by the pointing, acquisition and tracking system (PAT) of the laser communication terminal (LCT). Such a LCT has already been in use in orbits since several years [2,3].

The first European commercial system providing optical intersatellite communications is the upcoming European Data Relay System (EDRS). This network consists of several satellites in different orbits and allows laser links over distances of up to 45,000 km with data rates of up to 1.8 Gb/s [4]. The core technology consists of the LCT which enables optical coherent transmission between satellites with binary phase shift keying (BPSK). To ensure homodyne detection, an optical phase-locked loop (OPLL) is used to adjust the frequency and phase of the local oscillator (LO) to the incoming data signal. The OPLL is based on a Costas-loop which is characterised by a nonlinear behaviour that influences the transmission performance and which can be described by a nonlinear differential equation (NLDE) system.

In this paper we investigate the OPLL based carrier recovery system and its nonlinear characteristic. We present the functionality of a phase-locked loop with optical coherent detection and the influence of different noise sources on the frequency acquisition process in a free-space optical (FSO) channel. The remainder of the paper is structured as follows: Section 2 gives an introduction to the OISL system. In section 3 the Costas-loop based carrier recovery is described. Section 4 contains the analysis of the nonlinear OPLL behaviour. Finally in section 5, we explain different noise sources and the cycle slip phenomena.

II. OPTICAL INTERSATELLITE LINK

Optical (inter-)satellite communication describes data transmission between satellites using laser sources in the near-infrared spectrum instead of the conventional radio frequency (RF). Fig. 1 illustrates a typical OISL scenario. As part of an earth observation process a satellite in a low-earth orbit (LEO) accumulates a high amount of data. This data should be transmitted to a ground station (GS) on earth in real time, e.g. in case of a fast tsunami forecast system. However, the transmission time window of a LEO satellite is too short (several minutes per day) and the RF link data rate is too low (~Mb/s) in order to send all data to GS during one flyover.

Fig. 1. Conventional RF scenario (left), OISL scenario (right)
Hence, real-time data transmission is not possible. Instead, if we assume the data would be available on a satellite in geostationary orbit (GEO) the time window would be large (24 hours per day). However, the data need to be transmitted fast from the LEO to the GEO satellite. Therefore, an optical link between the satellites with data rates in the range of several Gb/s is used.

Compared to terrestrial optical communication systems, OISL has several differences. First, the equipment has to fulfill different quality requirements. Due to its location in space all parts have to be space-qualified, e.g. in terms of radiation and temperature hardness. Second, the narrow beam width needs line-of-sight between the satellites, so a complex PAT system is needed. Third, once established in space, the system must run for e.g. 15 years in a geostationary orbit without any maintenance. Finally, a significant difference is the in orbit FSO channel, which is less complex than e.g. the optical fiber channel. This is different for FSO transmission through the atmosphere [5]. Fig. 2 illustrates the block diagram of a typical OISL-BPSK transmission system. Usually both, transmitter and receiver part, are implemented in one LCT for each satellite in order to guarantee bidirectional transmission. Typical laser source is an optical pumped neodymium-doped yttrium aluminium garnet (Nd:YAG) laser at 1064 nm. The light is externally modulated by a phase-modulator (PM), driven by the electrical data signal and amplified up to 2 W by an ytterbium-doped fiber amplifier (YDFA). After transmission through the free-space channel the received optical signal is detected by the coherent receiver, containing a 90° optical hybrid, which superimposes the receive light with the LO light. The resulting I- and Q-signal is given as

\[
\begin{align*}
U_I(t) &= G R \tilde{s}_1 \tilde{s}_2 \cos (\varphi(t) + \phi_0) \\
U_Q(t) &= G R \tilde{s}_1 \tilde{s}_2 \sin (\varphi(t) + \phi_0),
\end{align*}
\]

where \(G\) denotes the gain of the transimpedance amplifier (TIA), \(R\) the responsivity of the photodiode and \(\varphi(t)\) the phase error. To ensure a correct frequency acquisition the control signal of the LO, i.e. the VCO, should only contain the phase error information and no data. In order to get such a data-free error signal \(U_I\) and \(U_Q\) are multiplied which results in

\[
U_{I_2}(t) \text{ is transformed to baseband by multiplying with a local oscillator } s_2(t) \text{ (synchronous demodulation). The local oscillator is controlled by an error signal } x(t) \text{ and } e(t) \text{ and therefore realized as a voltage controlled oscillator (VCO). The two signals are described as follows:}
\]

\[
s_1(t) = \tilde{s}_1 \sin \left( \omega_0 t + \phi_1(t) + \phi_{bo} \right) \quad \text{with} \quad \phi_1(t) = \Delta \omega(t) t + \phi_{bo} \\

s_2(t) = \tilde{s}_2 \cos \left( \omega_0 t + \phi_2(t) \right) \quad \text{with} \quad \phi_2(t) = K_\Delta \int x(t) dt + \phi_{bo},
\]

where \(\tilde{s}_1, \tilde{s}_2\) denote the signal amplitudes, \(\omega_0\) the carrier frequency, \(\Delta \omega(t)\) the time varying frequency offset due to the Doppler shift, \(K_\Delta\) the frequency gain of the VCO in Hz/V, \(\phi_{bo}\) the phase modulation, \(\phi_{bo}\) and \(\phi_{bo}\) a constant phase offset.

III. COSTAS-LOOP BASED CARRIER RECOVERY

The presented carrier recovery is a Costas-loop based control system as shown in Fig. 3. The incoming data signal

Fig. 2. LCT setup of an optical intersatellite communication system

Fig. 3. Basic structure of a Costas-loop based carrier recovery
In case of BPSK, i.e. $\phi_d(t) \in [0, \pi]$, the error signal results in

$$
\varepsilon(t) = K_p \sin(2\varphi(t)),
$$

(4)

where $K_p = (GR_0 \hat{s}_2)^2/2$ denotes the phase discriminator gain. The error signal $\varepsilon(t)$ contains the residual frequency offset. The signal is passed to the loop filter, which describes the control element of the loop and defines the loop dynamic. Typical loop filters are the lead-integrator (active) filter and the lead-lag (passive) filter, with the following transfer functions:

$$
F_{\text{active}}(s) = \frac{1 + sT_1}{sT_2}, \quad F_{\text{passive}}(s) = \frac{1 + sT_1}{1 + sT_2},
$$

(5)

where the time constants $T_1$ and $T_2$ influence the loop dynamic. Finally, the control signal $x(t)$ adjusts the frequency of the fast tuneable LO based on the information of the error signal. Fig. 5 shows a typical frequency lock-in process in an optical intersatellite link with active loop filter observed at the phase error signal $\varepsilon(t)$ and the frequency error (with $K_0=5.2 \text{ MHz/V}$, $K_D=0.79 \text{ V}$, $T_1=0.21 \mu\text{s}$, $T_2=1.2 \mu\text{s}$). The frequency offset between the incoming signal and the LO at the beginning of the lock-in process was set to 5 MHz. In this example the OPLL locks after ~8 µs. It should be noticed, that both signals are already overlapped by noise. Typical noise sources will be described in section 5. As seen in (4) the OPLL has a sinusoidal nonlinear behaviour, which will be described in detail in section 4.

IV. ANALYSIS OF THE NONLINEAR OPLL

In order to investigate the nonlinear characteristic of the presented OPLL we develop, in a first step, the equivalent mathematical block diagram of a nonlinear Costas-based optical phase-locked loop. Fig. 6 shows the resulting control loop which is based on (4).

$$
\phi(t) = \phi(t) - \phi(t) = \phi(t) - K_p \int x(t) \, dt.
$$

(9)

Deriving the phase error gives

$$
\frac{d\varphi}{dt} = \frac{d\phi}{dt} - K_p x(t),
$$

(10)

and the resulting control signal can be described as

$$
x(t) = -\frac{1}{K_p} \left[ \frac{d\varphi}{dt} - \frac{d\phi}{dt} \right].
$$

(11)

Inserting (4) and (11) in (7) results in

$$
-\frac{1}{K_p} \left[ \frac{d\varphi}{dt} - \frac{d\phi}{dt} \right] \left[ \frac{d\varphi}{dt} + \frac{1}{T_2} \right] = K_p \sin(2\varphi) \left[ \frac{d\varphi}{dt} + \frac{1}{T_1} \right] m.
$$

(12)

After rearranging (12) the nonlinear differential equation is

$$
\frac{d\varphi}{dt} - \frac{d\phi}{dt} + \phi \frac{1}{T_2} - \phi \frac{1}{T_2} = -K \left[ 2\phi \cos(2\varphi) + \sin(2\varphi) \right] \frac{1}{T_1},
$$

(13)

with $K = K_p K_D m$. 

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**Fig. 4.** Optical phase-locked loop based on Costas-loop implementation

**Fig. 5.** Lock-in process: phase error (left) and frequency error (right)

**Fig. 6.** Equivalent mathematical block diagram of a nonlinear OPLL

We assume a passive loop filter $F(s)$ as follows

$$
F(s) = \frac{1 + sT_1}{1 + sT_2} = \frac{1}{m} - \frac{1}{1 + sT_2} = \frac{X(s)}{E(s)}, \text{ with } m = \frac{T_1}{T_2}.
$$

(6)

Transforming (6) into time domain results in

$$
\varepsilon(t) \left[ \frac{d\varepsilon}{dt} + \frac{1}{T} \right] = \phi(t) \left[ \frac{d\phi}{dt} + \frac{1}{T} \right] m.
$$

(7)

The LO, i.e. the VCO, adjusts the frequency according to the input voltage with gain $K_0 \text{ [Hz/V]}$. The corresponding phase change can be observed by integrating the input signal

$$
\phi(t) = K_0 \int x(t) \, dt.
$$

(8)
According to (1) and assuming that $\Delta \omega$ keeps nearly constant during lock-in process, the following can be deduced:

$$\frac{d\Delta \omega}{dt} = \Delta \phi = \Delta \omega, \quad \frac{d\phi}{dt} = 0.$$  \hspace{1cm} (14)

Finally, the NLDE system results in

$$\begin{align*}
\frac{d\phi}{dt} &= \phi \\
\frac{d\omega}{dt} &= -\phi \left[ \frac{1}{T_2} + 2K \cos(2\phi) \right] - K \frac{1}{T_1} \sin(2\phi) + \frac{\Delta \omega}{T_2}.
\end{align*}$$  \hspace{1cm} (15)

An analytical solution of (15) does not exist. Therefore, a numerical approximation is required in order to investigate the nonlinear behaviour. A common graphical method to investigate this approximation of the nonlinear PLL is the phase plane diagram. This diagram shows the frequency error $d\phi/dt = \phi$ depending on the phase error $\phi$. The phase error is wrapped in the range $\pm \pi$. Each point $(\phi, \dot{\phi})$ on the phase plane is part of a specific solution of the NLDE system. A solution is a so called trajectory.

As well-known, the PLL has a specific working range, i.e. the lock-in range $\Omega_L$ and the hold-in range $\Omega_H$ ($\Omega_L < \Omega_H$), depending on the loop parameters. Within the lock-in range all frequency offsets, $\Delta \omega < \Omega_L$, can be compensated for. The high initial frequency offset at the beginning of a transmission should be inside this range. Offsets between lock-in and hold-in range ($\Omega_L < \Delta \omega < \Omega_H$) cannot be compensated for. However, if the PLL is already locked it will keep the lock status within this range unless the frequency changes are too fast (i.e. no prompt changes). Outside of the lock-in range lock-in is impossible ($\Delta \omega > \Omega_H$). According to [6] the ranges based on a 2nd order PLL, which are valid for the 2nd order Costas-loop as well, with passive filter can be calculated as follows:

$$\Omega_H = \pm \frac{K_{\phi}K_{\omega}}{2m - m^2}, \quad \Omega_L \approx \Omega_H \sqrt{2m - m^2}.$$  \hspace{1cm} (16)

The above mentioned statements concerning lock-in and hold-in range can be verified by looking at the phase plane diagram for different frequency offsets $\Delta \omega$ in (15), i.e. the frequency offset between the two lasers. The NLDE system has an infinite number of solutions, so infinite trajectories exist. We present a few of them in the phase planes. Fig. 7 shows the phase plane in case of $\Delta \omega < \Omega_L$. It can be observed that all trajectories will end up in a stable lock-in point P, i.e. the loop will always lock for each starting point $(\phi_0, \dot{\phi}_0)$.

Fig. 8 shows the phase plane in case of $\Omega_L < \Delta \omega < \Omega_H$. First of all, it can be seen that a stable periodic state exists with a residual frequency offset. Most of the trajectories will approximate to that state. Secondly, some trajectories will still end up in a stable point P. Finally, it is possible that phase error changes will cause the loop to lose the lock. If we assume the actual state of the loop is a point beneath a separatrix (a special trajectory which separates the path of the residual trajectories and which starts or ends in an unstable point Q), phase noise can move this point to a state above the separatrix and therefore unlocks the loop.

Finally, in case of $\Delta \omega > \Omega_H$ the phase plane is shown in Fig. 9. As expected no solution exists which will end in a stable point P, the OPLL will not lock. It should be mentioned, that the above presented phase plane examples only show some few trajectories and only for positive frequency offsets. The behaviour would be same for negative ones.
The above mentioned noise-induced phase error changes, which may unlock the loop, can additionally cause another effect, even if the frequency offset is inside the lock-in range. The so called cycle slip phenomena.

V. CYCLE SLIP INVESTIGATION

As already described in section III the signals $U_i$ and $U_Q$ contain the demodulated data. These signals get mainly distorted by two noise sources, the phase and shot noise. The demodulated signal overlapped with the two noise sources is

$$U_{\text{Data}}(t) = U_i(t) + jU_Q(t)$$

where $\varphi_{\text{PSN}}$ denotes the phase noise, $U_{PD}$ the beating amplitude and $U_{SN}$ the intensity noise due to shot noise.

As typical in optical (satellite) transmission systems the laser phase noise is one of the main signal impairments. Current OiSL systems as part of EDRS include lasers with a linewidth of approx. 10 kHz [2]. Due to the specific linewidth $\Delta \nu$ both, the transmitter laser as well as the local oscillator, cause phase rotation which influences the signal quality. The phase noise between two subsequent phase values with time difference $\tau$ can be expressed as a Gaussian noise process with variance

$$\sigma_{\text{PSN}}^2 = 2\pi \Delta \nu \tau .$$

(18)

Additionally, shot noise is induced in the photodiode caused by the stochastic arrival of photons. It can be modelled as zero-mean Gaussian random process with variance

$$\sigma_{\text{SN}}^2 = 2qI_{\text{PD}}(t)B_s .$$

(19)

where the electric charge $q = 1.6 \times 10^{-19}$ As, $I_{PD}$ denotes the photo current before TIA and $B_s$ the single-sided receiver bandwidth. Both, phase noise as well as shot noise, influence the phase error $\varphi$. Regarding the phase plane, high phase error changes can activate a different trajectory which may end in a new stable point. In that case the loop will lose the lock status and lock-in again after several of $\pm \pi$ phase jumps (i.e. cycle slips), depending on the activated trajectory on which the phase error will move. However, in the worst case, the noise induced phase error change will cause the loop to lose the lock status if the activated trajectory ends in the stable periodic state, as described in the previous section. Fig. 10 shows the probability density function of the phase error after time $T_1 < T_2 < T_3$. If considering only one simulation ($n = 1$), the distribution randomly moves either to the right or to the left (here to the right). The lower subfigure of Fig. 10 shows the distribution in case of repeating this simulation more than once ($n = 500$). Now the phase error is symmetrically distributed.

Usually cycle slips cause two impairments. First of all, as described above, cycle slips produce phase jumps of multiple $\pm \pi$, hence, the demodulated data signal will experience phase jumps which result in bit errors (in case of PSK). This problem can be solved by using differential decoding. Secondly, each cycle slip event produces burst errors at the decision element, since the phase jump process, i.e. the phase rotation, is not immediately but has a finite duration. Regarding BER measurements burst errors should be avoided and hence cycle slips should be compensated for, e.g. by optimisation of the OPLL bandwidth.

VI. CONCLUSIONS

We presented an analysis of the nonlinear OPLL, which is used for carrier recovery in optical intersatellite links. The Costas-loop based control system is required for coherent detection in order to recover amplitude and phase information. The corresponding NLDE system was derived and graphically investigated by phase plane diagrams. Depending on the initial frequency offset as well as the noise influence trajectories will end either in a stable lock point or in a stable but unlocked periodic state. Typical noise sources were explained as well as the cycle slip phenomena which influences the system performance in optical intersatellite communication systems.

REFERENCES