Fiber Nonlinearity Compensation for Dispersion Unmanaged PDM 8-QAM CO-OFDM using Expectation Maximization

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Abstract

Fiber nonlinearity compensation via expectation maximization is investigated for PDM CO-OFDM. An improvement in Q-factor of 0.33 dB is observed for 1400-km 100 Gb/s single channel and 1.44 dB for 1000-km 700 Gb/s WDM transmission.

I. INTRODUCTION

Fiber nonlinearity (NL) compensation in coherent optical OFDM (CO-OFDM) systems is a key technique for achieving the channel capacity. Thus, effective algorithms are being investigated specified for CO-OFDM systems such as DFT-spread technique [1], intensity based SPM compensation [2] and digital back-propagation [3]. Nevertheless, these algorithms require major modifications of the transceivers or *a priori* knowledge of the fiber link. Recently, an iterative method named expectation maximization (EM) was discussed enabling an effective and adaptive fiber NL compensation for polarization-division multiplexing (PDM) single-carrier transmission with dispersion-managed links [4].

In this paper, the effectiveness of the EM algorithm is further investigated for single channel (S-Ch) and wavelength-division multiplexing (WDM) PDM CO-OFDM long haul transmission with dispersion-unmanaged link. Simulations based on 100 Gb/s S-Ch and 700 Gb/s WDM PDM transmissions are evaluated. The EM algorithms is applied and combined with the RFP technique, which is an adaptive method used for fiber NL mitigation [5] as well as laser phase noise (PN) mitigation [6]. Finally, a joint digital signal processing structure with both RFP and EM algorithm is proposed, which effectively improves the fiber NL tolerance.

II. EXPECTATION MAXIMIZATION

Let M-QAM be the modulation format used in the transmitter and $p(k)$ is the probability density function (PDF) of the kth constellation points being transmitted. Suppose the conditional PDF of the received $\boldsymbol{y}$ (bold letter indicates 2-D vector) given that the kth constellation point is transmitted can be expressed as:

$$p(\boldsymbol{y}\,|\,k) = N(\boldsymbol{y}\,|\,\boldsymbol{\mu_k}, \Sigma_k) \tag{1}$$

where $N(\boldsymbol{y} \mid \boldsymbol{\mu_k}, \Sigma_k)$ represents the bivariate Gaussian PDF defined as:

$$N(\boldsymbol{y}\,|\,\boldsymbol{\mu_k}, \Sigma_k) = \frac{1}{2\pi\sqrt{|\Sigma_k|}} e^{-\frac{1}{2}(\boldsymbol{y}-\boldsymbol{\mu_k})^T \Sigma_k^{-1} (\boldsymbol{y}-\boldsymbol{\mu_k})} \tag{2}$$

where $|\bullet|$ stands for determinant, $\boldsymbol{\mu_k}$ is the mean value and Σ_k is the covariance matrix given as:

$$\Sigma_k = \begin{bmatrix} \sigma_{I,k}^2 & \rho\sigma_{I,k}\sigma_{Q,k} \\ \rho\sigma_{I,k}\sigma_{Q,k} & \sigma_{Q,k}^2 \end{bmatrix} \tag{3}$$

where $\sigma_{I,k}^2$, $\sigma_{Q,k}^2$ are the Gaussian noise variance along the inphase (I) and quadrature (Q) component and ρ is the correlation factor. Thus, the PDF of each received $\boldsymbol{y}$ is given as a mixture of Gaussian (MoG):

$$p(\boldsymbol{y}) = \sum_{k=1}^{M} p(k) N(\boldsymbol{y}\,|\,\boldsymbol{\mu_k}, \Sigma_k). \tag{4}$$

For instance, suppose ρ=0, $\sigma_{I,k}^2=\sigma_{Q,k}^2$, each cluster (defined as the grouping of spread symbols around a mean value) in the received constellation is circularly-symmetric distributed.

If the phase distortion is introduced due to the fiber NL and laser PN, each cluster becomes none circularly-symmetric Gaussian distributed, i.e. $\rho \neq 0$ and/or $\sigma_{I,k} \neq \sigma_{Q,k}$. In order to capture the PDF of the distorted received constellation, the most likely $\boldsymbol{\mu}=[\boldsymbol{\mu_1},...,\boldsymbol{\mu_M}]$ and $\Sigma=[\Sigma_1,...,\Sigma_M]$ is required to maximize the log-likelihood of the received constellation. Assuming all the received constellation points are independent identically distributed and total N_{sym} points are received, $\boldsymbol{\mu}$ and Σ is given as:

$$\begin{aligned}[\mu\ \Sigma] &= \operatorname{argmax}\{\log[p(\boldsymbol{Y})]\} \\ &= \underset{\substack{\boldsymbol{\mu}=[\boldsymbol{\mu_1}\cdots\boldsymbol{\mu_M}] \\ \Sigma=[\Sigma_1\cdots\Sigma_M]}}{\operatorname{argmax}} \left\{ \sum_{n=1}^{N_{sym}} \log \sum_{m=1}^{M} p(\boldsymbol{x_m}) N(\boldsymbol{y_n}\,|\,\boldsymbol{\mu_m}, \Sigma_m) \right\}\end{aligned} \tag{5}$$

where $\boldsymbol{Y}=\{\boldsymbol{y_n}|n=1,...,N_{sym}\}$, $\log[p(\boldsymbol{Y})]$ is the log-likelihood of all received constellation points.

The EM algorithm is an iterative method that uses the received constellation points (unknown data) to compute the most likely $\boldsymbol{\mu}$, Σ and is separated into E-step and M-step. In the E-step of the ith iteration, the *a posteriori* probability of each received symbol belonging to the kth constellation point γ_{nk}^i is calculated as:

$$\gamma_{nk}^i \equiv P_i(k\,|\,\boldsymbol{y_n}) = \frac{\boldsymbol{\pi_k}^{i-1} N(\boldsymbol{y_n}\,|\,\boldsymbol{\mu}_k^{i-1}, \Sigma_k^{i-1})}{\sum_{m=1}^{M} \boldsymbol{\pi_m}^{i-1} N(\boldsymbol{y_n}\,|\,\boldsymbol{\mu}_m^{i-1}, \Sigma_m^{i-1})} \tag{6}$$

where $\boldsymbol{\pi_k}$ is the mixing factor ($\boldsymbol{\pi_k}=1/M$, when the constellation points of M-QAM are uniformly transmitted).

By means of γ_{nk}^i, $\boldsymbol{\mu}$ and Σ are updated in the M-step of the ith iteration [7] according to equation group (7):

$$\begin{aligned} N_k^i &= \sum_{n=1}^{N_{sym}} \gamma_{nk}^i, \qquad \boldsymbol{\pi}_k^i = \frac{N_k^i}{N_{sym}}; \\ \boldsymbol{\mu}_k^i &= \frac{1}{N_k^i} \sum_{n=1}^{N_{sym}} \gamma_{nk}^i \boldsymbol{y_n}; \end{aligned} \tag{7}$$

$$\Sigma_k^i = \frac{1}{N_k^i}\sum_{n=1}^{N_{sym}} \gamma_{nk}^i (\boldsymbol{y}_n - \boldsymbol{\mu}_k^{i-1})(\boldsymbol{y}_n - \boldsymbol{\mu}_k^{i-1})^T. \quad (7)$$

Therefore, $\log[p_i(\boldsymbol{Y})]$ in the ith iteration can be calculated according to (5) via the trained $\boldsymbol{\mu}^i$ and Σ^i in the ith iteration that consist of all the $\boldsymbol{\mu}_k^i$ and Σ_k^i. This iterative procedure will be ended when the following condition is fulfilled:

$$\frac{\log[p_i(\boldsymbol{Y})] - \log[p_{i-1}(\boldsymbol{Y})]}{\log[p_i(\boldsymbol{Y})]} < \varepsilon \quad (8)$$

where ε is the predefined maximum error tolerance.

III. Simulation Setup and Results

The simulations are based on a 100 Gb/s S-Ch of 1400 km and a 1000-km 7×100 Gb/s WDM PDM 8-QAM CO-OFDM transmission. For S-Ch transmission, the two polarization (Pol) signals are generated via the standard OFDM transmitter (TX). Each OFDM TX uses an FFT size of 2048, among which 1770 subcarriers are modulated with 8-QAM and the rest are zero padded. The length of the cyclic prefix is 140 samples. Training symbols (TSs) are inserted every 26 OFDM symbols for channel equalization. For WDM transmission, 7 OFDM bands are multiplexed with a channel spacing of 50 GHz. The DAC is operating at 21 GS/s. The FEC overhead is set to 7% targeting an FEC limit of 10^{-3}.

The fiber link is composed of 14 spans for S-Ch transmission and 10 spans for WDM transmission. Each span consists of 100 km SSMF and an EDFA. ASE noise is launched in each stage EDFA. The parameters of the SSMF are given as: α=0.2 dB/km, γ=1.2 $W^{-1}km^{-1}$, D=17 ps/nm·km, S=0.021 ps/nm^2·km and PMD=0.05 ps/√km. The ECL lasers in the transceivers have a laser linewidth (LW) of 100 kHz.

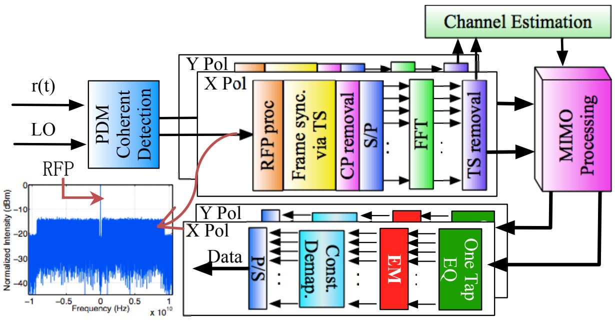

Fig. 1. PDM OFDM receiver structure. LO: local oscillator; sync.: synchronization; MIMO: Multiple Input Multiple Output; Const. Demap.: constellation demapping;

The receiver structure is shown in Fig. 1. The EM algorithm is applied after the one tap equalizer (EQ). The RFP processing is optimized refer to [5]. The performance is represented by Q factor derived from the bit error rate (BER) which is counted out of 2×10^6 bits.

Fig. 2(a) illustrates the results of the S-Ch transmission. The optimum Q factor with RFP method only is 10.35 dB at a launch power (LP) of -0.73 dBm, while the joint performance provided by the RFP+EM method increases the Q factor to 10.68 dB at LP of -0.73 dBm. An improvement of 0.33 dB is observed.

As shown in Fig. 2(b), due to the cross-phase modulation, EM offers more significant system improvement for WDM channel transmission. Here, the joint method further increases the Q factor with 1.44 dB compared with applying only RFP method.

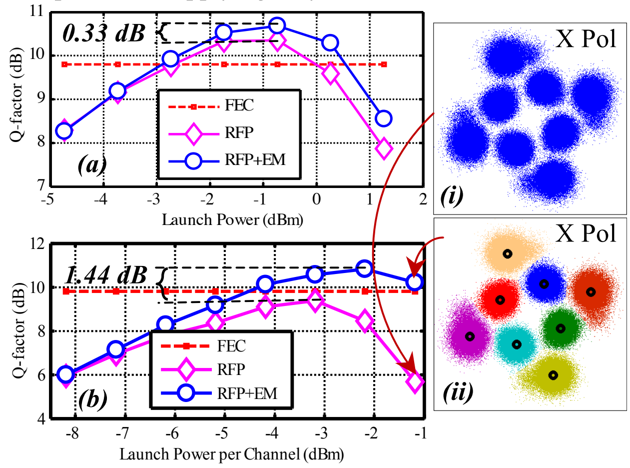

Fig. 2: Simulation results of S-Ch transmission (a) and WDM (b) transmission. Inset (i) The received constellation of WDM transmission at LP/Ch -1.19 dBm; Inset (ii) The decision region optimized via EM.

Inset (i) of Fig. 2 shows the received constellation of the WDM transmission in X Pol at a LP = -1.19 dBm for each channel without EM method. The phase noise introduced common phase rotation and elliptical shape clusters are clearly illustrated. Due to the fact that TSs are added every 26 OFDM symbols and the fiber NL phase distortion evolves with the instantaneous signal power, the one tap equalizer fails to correct the phase distortion estimated by the TSs. Besides, the performance of the RFP method is also limited due to the low pass filter applied to extract the pilot which indicates that the RF-pilot only tracks the fiber NL distortion with a limited bandwidth. Therefore, after RFP processing there is residual NL phase noise left being uncompensated.

Inset (ii) illustrates the most likely MoG with the trained optimal $\boldsymbol{\mu}$ (indicated by ○) and Σ using EM. The distribution of the received constellation described via MoG enables a simplified maximum-likelihood symbol detection and in the digital signal processing the received constellation points with the same color are de-mapped into the same transmitted constellation symbols, through which the fiber NL phase noise can be effectively compensated.

IV. Conclusions

Simulation results show that, compared with only RFP technique, a joint RFP+EM method achieves an increase in Q factor of 0.33 dB for 1400-km 100 Gb/s S-Ch and 1.44 dB for 1000-km 700 Gb/s WDM PDM 8-QAM CO-OFDM transmission, respectively.

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