Is Tomlinson-Harashima Precoding Suitable for Fiber-Optic Communication Systems?

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Summary / Abstract

Tomlinson-Harashima precoding (THP) is a pre-compensation method, which is exactly fitted to the channel impulse response in order to completely compensate for its linear impairments, and it is in-fact a generalization of partial response precoding. THP is very popular in the field of wireless communications, namely in multi-input multi-output (MIMO) scenarios. It offers an alternative to the decision-feedback equalizer (DFE) with the possibility to apply channel coding conjunctively.
In this paper, the compatibility of THP with fiber-optic communication systems is investigated. Different configurations of THP are considered and compared with DFE by means of performance and complexity. Results are examined for 4QAM modulation, where the length of the transmission link and the number of equalizer coefficients are varied. In addition, the effect of the receiver filter's bandwidth on the performance of THP is evaluated. In the second step, possible scenarios for using THP in a fiber-optic system, as well as challenges, which result from properties of the system and equalizer, are addressed and discussed.

1 Introduction

In fiber-optic communication systems, transmission impairments can be mitigated using either pre- or post-compensation. In order to improve the performance of equalization, it is necessary to obtain both the magnitude and phase of the optical signal's electric field. Therefore, pre-compensation techniques have an advantage over post-compensation, in a sense that the magnitude and phase information is already available at the transmitter side. One well-known example is Duobinary partial response transmission, which is used to increase the tolerance of the optical data signal to dispersion [1], [2].
Tomlinson-Harashima precoding (THP) is a pre-compensation method, which is exactly fitted to the channel impulse response in order to completely compensate for its linear impairments, and it is in-fact a generalization of partial response precoding. Invented independently and almost simultaneously by M. Tomlinson in 1971 [3] and H. Harashima in 1972 [4], THP offers an alternative to the decision-feedback equalizer (DFE) with the possibility to apply channel coding *conjunctively* [5]. THP is very popular in the field of wireless communications. A wireless communication system, which consists of base-stations and spatially distributed users, can be considered as a multiple-input multiple-output (MIMO) channel. For this multiuser scenario it can be shown that a system with precoding applied at the base-station performs significantly better than a system, where post-processing is applied for each user-receiver individually [6]. In addition, by keeping the entire compensation load at the base-station, the complexity of the mobile wireless units can be reduced, which is important e.g. for the unit's battery life. Apart from mobile communications, THP is defined in the standard for 10GBASE-T Ethernet over twisted-pair cables, where a 128 symbols double square quadrature (DSQ128) modulation is used together with low-density parity-check (LDPC) channel coding [7].
Lately, many well-known compensation algorithms from the field of digital communications, and namely wireless communications, are adopted to be used in fiber-optic communication systems. This is possible thanks to the progressive research to develop faster and faster electronic circuits [8] and the availability of coherent detection. THP, however, and to the best of our knowledge, hasn't yet been purposed for pre-compensation of fiber-optic transmission impairments.
In this paper, the compatibility of THP with fiber-optic communication systems is investigated. The remaining of the paper is organized as follows: Section 2 is dedicated for theoretical background, where the principles and characteristics of THP are explained for optical and non-optical transmission systems. The simulation results are given in section 3. In section 4 possible scenarios for using THP in a fiber-optic system, as well as their challenges, are addressed and discussed. Section 5 concludes this paper.

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2 Theoretical Background

The purpose of the following section is to provide the reader with a background on conventional THP, and to specify the adjustments to be made in order to be able to efficiently use THP in fiber-optic transmission systems. This section opens with general motivation, followed by principles and important characteristics of THP in non-optical communication systems. Later on, the principles of THP for optical communication systems will be introduced, and the implementation requirements will be discussed.

For the subsections concerning a non-optical communication system, the discrete-time channel model illustrated in **Figure 1.a** is used, and defined as follows:

$$r_k = \sum_{l=0}^{N_h-1} h_l d_{k-l} + n_k\,, \tag{1}$$

where d_k is the data symbol sequence, h_k is the channel's impulse response of a finite length N_h, n_k is an additive white Gaussian noise (AWGN) sequence, and r_k is the received sequence, i.e. the received signal after sampling. The channel is assumed to be causal and its main cursor is $h_0 = 1$. Clearly, for $N_h > 1$ the received signal suffers from inter-symbol interference (ISI).

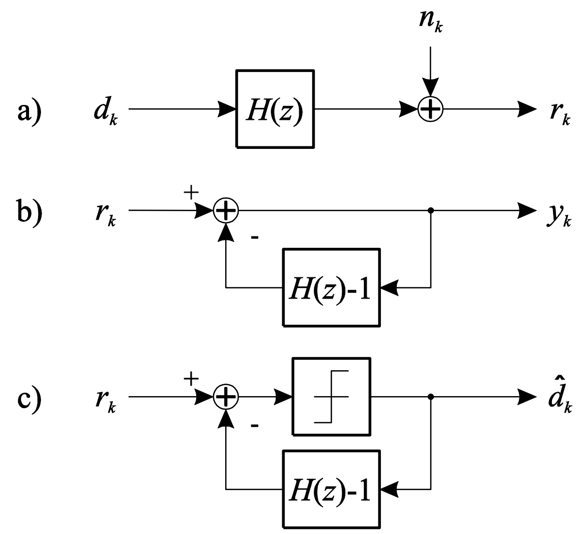

Figure 1 Discrete-time models of (a) an ISI-AWGN channel, (b) a zero-forcing IIR equalizer and (c) a decision-feedback equalizer.

2.1 Motivation

This subsection follows the motivation given in [5]. In order to address the equalization problem in a digital system, one can use zero-forcing equalization (ZFE), which can be achieved using either a finite impulse response (FIR) or an infinite impulse response (IIR) filter. While equalization with FIR is either suboptimal or requires a higher sampling frequency than the Baud rate for an exact solution [9], the IIR structure given in **Figure 1.b** can exactly realize the channel's inverse transfer function $1/H(z)$, where $H(z)$ is the Z transformation of h_k, i.e.

$$H(z) = \sum_{k=0}^{N_h-1} h_k z^{-k}\,. \tag{2}$$

In other words, exact equalization can be achieved for sampling at the Baud rate. Unfortunately, since communication channels are not necessarily *minimum-phase*, this structure turns to be unstable. Moreover, even for minimum-phase channels, noise enhancement is evident at the output of the equalizer, where for channels with zeros close to the unit circle the enhancement factor becomes prohibitive for a correct function of the system. In order to avoid instability and/or noise enhancement problems, a nonlinear device, namely a decision device or a "slicer", can be inserted in the upper arm of the feedback loop, as illustrated in **Figure 1.c**. This structure is well-known in the literature as decision-feedback equalization (DFE), which under the assumption of correct decisions removes completely the ISI and leaves the white noise uncolored DFE, on its down-side, has two main disadvantages: the first is the so-called *error propagation* effect, where wrong symbol decisions may lead to a series of symbol errors. The second and more important is the inability to *easily* incorporate DFE with channel coding techniques, such as trellis-coded modulation (TCM) or LDPC. Assuming linearity, these two disadvantages can be eliminated, if the IIR structure will be positioned at the *receiver side*. Similarly to DFE, a nonlinear device is used in order to limit the output power of the pre-compensator, and in that way to achieve stability, as will be seen in the following subsection.

2.2 Principles of THP

A block diagram of the system incorporating THP is given in **Figure 2**. As can be seen, the structure of the precoder is almost identical to that of the DFE, where instead of a slicer it uses a nonlinear device with a zero-mean modulo $2M$ function, which is defined as:

$$m(x) = \left((x+M) \bmod 2M \right) - M\,, \tag{3}$$

where M is even and stands for the number of constellation points in a bipolar M-ASK modulation. An example for the modulo function in case of $M = 4$ is illustrated in **Figure 3**. Since this operator forces the output of the precoder to the half-open interval $[-M,M)$, stability is ensured. The use of such a nonlinear operation raises the question of whether the precoded data can be successfully decoded at the receiver side. Fortunately, using a linearized model of THP, it can be proven that the same modulo operator as described before, when applied at the receiver side, can perfectly restore the original data sequence [5].

THP can be generalized for complex signals and/or channel impulse responses by applying the modulo operation on the real and imaginary components *separately*. In the

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following, the properties of THP and their implications on the communication system are discussed.

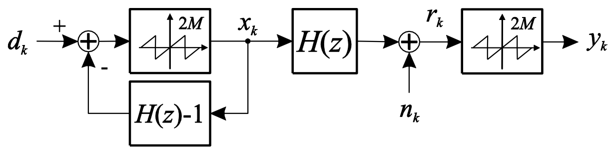

Figure 2 A communication system with THP.

2.2.1 Properties of the Transmit Signal

The precoding sequence x_k is characterized as almost white and uniformly distributed in the interval $(-M,M)$. This characteristic becomes more exact if M or the length of the channel impulse response increases. The first is fulfilled by using higher order modulation formats. Under these assumptions, the power of x_k is $M^2/3$ and $2M/3$ for bipolar M-ASK and M-QAM modulation formats, respectively [5]. By examining the power of the sequence before precoding: $(M^2-1)/3$ and $2M/3$ for bipolar M-ASK and M-QAM modulation formats, respectively, a *precoding loss* is evident. This loss affects the signal-to-noise ratio (SNR) performance of THP in comparison with that of DFE.

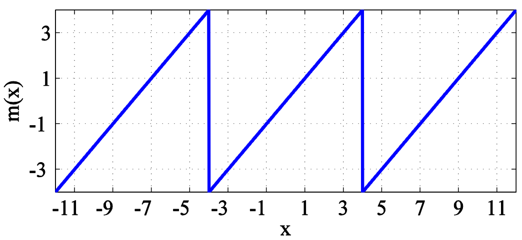

Figure 3 A zero-mean *2M modulo* function for a 4-ASK modulation format ($M = 4$).

2.2.2 Properties of the Received Signal

Due to the modulo operation at the transmitter side, the received signal (neglecting the noise) can be defined as:

$$r_k = d_k + p_k\,, \tag{4}$$

where $p_k \in 2M\mathbb{Z}$. This phenomenon is determined in [5] as an *extended signal set*. This set is mapped to a periodic extension of the original signal constellation for purposes of successful decoding. For complex constellations periodicity has to be fulfilled in both dimensions of the complex plain. Therefore, cross-shaped and circular constellations, as opposed to M-QAM, cannot be applied with THP. Regarding the dynamic range of the received signal, it is dependent on both M and the sum over the magnitude of the channel coefficients. Therefore, it can become very large and result in a high received power and will require a higher resolution of the analog-to-digital converters (ADC) and DSP units.

2.3 Principles of THP in Fiber-Optic Communication Systems

The THP structure shown before, similarly to DFE, can be applied efficiently for causal channels, i.e. for channels with postcursor ISI only. The optical communication channel, however, has in fact a non-causal complex impulse response with both postcursor and precursor ISI components, where an example can be seen in **Figure 4** for a raised-cosine-in-time-domain pulse-shaping filter and 400 km of standard single mode fiber (SSMF). The output of the discrete-time channel can be written as follows:

$$r_k = h_0 d_k + \sum_{i=-\infty}^{-1} h_i d_{k-i} + \sum_{i=1}^{\infty} h_i d_{k-i} \tag{5}$$

where the first term is the desired output, and the second and third terms are the unwanted pre- and postcursor ISI, respectively. The feedback equalizer (FBE) is capable of removing the postcursor ISI by employing THP. However, in order to efficiently compensate for precursor ISI, it is necessary to use a feed-forward equalizer (FFE) in conjunction with THP [9].

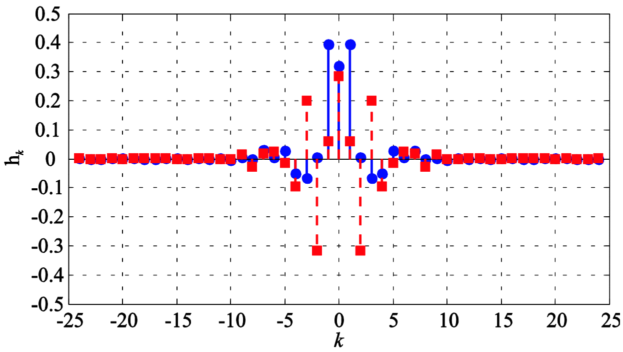

Figure 4 Real (blue circle) and imaginary (red square) components of a sampled impulse response of a system with a raised-cosine-in-time-domain pulse shaping filter and 400 km of SSMF

A system performing this task is shown in **Figure 5**. As can be seen, an FFE $F(z)$ is chosen to be positioned after the channel $H(z)$, although it can be positioned before as well. In order to derive the optimum filter coefficients, it is possible to use the well-known minimum mean square error (MMSE) algorithm [10].

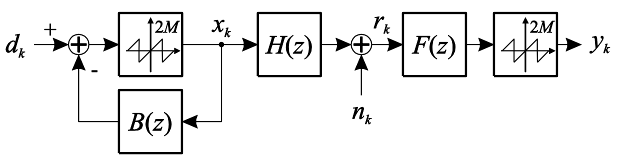

Figure 5 A communication system with THP and an FFE for effective compensation of pre- and post-cursor ISI.

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2.3.1 MMSE THP

An Optimum receiver (by means of MMSE) can be derived for an ISI channel with AWGN using an MMSE-FFE-DFE [10]. In case of THP the feedback filter (equivalent to the DFE) is placed at the transmitter side. It can be shown that the nonlinear modulo operation does not affect the system performance and can be modelled as an additive sequence to force x_k to the half-open interval $(-M,M)$ [5]. Therefore, the derivation for the optimum filter coefficients for THP is the same as for DFE, with the exception that the data signal to be considered for the MMSE criterion is the precoded sequence x_k and not d_k. Nevertheless, for large constellations, where the precoding loss is negligible, the difference is almost unnoticeable. In addition, Since the MMSE solution approaches the zero-forcing (ZF) solution for high SNR values, the difference between MMSE-DFE and MMSE-THP is only noteworthy for low SNR.

Using a derivation of the MMSE equalizer that uses the knowledge of the channel impulse response, the same algorithm used for MMSE-DFE can be used for MMSE-THP, with the exception mentioned above. The optimum coefficients of the feed-forward and feedback filters are

$$\mathbf{f} = \left(\mathbf{H}^{\dagger}\mathbf{P}\mathbf{H} + \lambda\mathbf{I}\right)^{-1}\mathbf{H}^{\dagger}\mathbf{e}_{\delta}\,;\ \mathbf{b} = \mathbf{M}\mathbf{H}\mathbf{f}\,, \qquad (6)$$

where $\mathbf{H}$ is the $(N_h + N_f - 1)\times N_h$ channel convolution matrix, $\mathbf{e}_\delta$ is the $N_f \times 1$ Dirac vector ($e_\delta = 1$), and $(\cdot)^{\dagger}$ is the conjugate-transpose operator. In addition:

$$\lambda = \sigma_n^2 / \sigma_x^2\,, \qquad (7)$$

$$\mathbf{M} = \left[\mathbf{0}_{N_b\times\delta} \quad \mathbf{I}_{N_b\times N_b} \quad \mathbf{0}_{N_b\times N_h - N_b - \delta}\right], \qquad (8)$$

and

$$\mathbf{P} = \mathbf{I} - \mathbf{M}^T\mathbf{M}\,, \qquad (9)$$

The MMSE-FIR-DFE solution does not only depend on the number of filter coefficients (N_f and N_b for the feed-forward and feedback filter, respectively), but on δ, the delay of the main cursor of the equalized signal, as well. In this work, an exhaustive search was used in order to find the optimal delay.

2.3.2 Implementation Requirements

THP requires the following, in order to be implemented as part of a fiber-optic system: First, the implementation of the feedback transmit filter is of significance, since it has to fulfill the optical signal Baud rate requirements. Second, since the precoded sequence is evenly distributed over $(-M,M)$ it is desired to have a digital-to-analog converter (DAC) with a high resolution, as similarly needed for OFDM transmission systems. Third, due to the fact that the optical channel's impulse response is complex, the resulting precoded sequence is complex as well, whether the data sequence is complex or not. Therefore, there is a need for an I-Q modulator for transmission. In addition, a modulo function has to be implemented both at the transmitter and receiver. Using the two's-complement representation of the words in the digital circuit, the modulo operation can be implicitly performed by using a certain number of bits that represent the integer. This was already proposed by Tomlinson as *modulo arithmetic* [3]. Last but on least, the channel information has to be known at the transmitter side. One way to achieve that is using a backward channel, where information about the channel characteristics will be transmitted from the receiver back to the transmitter in order to apply THP. That way, channel estimation can be performed at the receiver side, and the information can be relayed to the transmitter. In case the channel is reciprocal, a training sequence can be sent from the receiver in order to estimate the channel at the transmitter. This way, quantization errors are avoided by relaying the estimation of the channel coefficients to the transmitter. It is also possible to apply THP even if only reduced channel state information is available at the transmitter. One can use statistical channel knowledge in order to determine the equalizer coefficients. Nevertheless, there are certain penalties that limit the transmission performance, and can be found in detail in [11].

3 Simulation Results

3.1 Simulation Setup

The simulation setup is shown **Figure 6**. A De-Bruijn pseudo random quaternary sequence was mapped to 4QAM symbols. The symbols d_k were processed with THP to yield the precoded symbols x_k. The symbol rate R_s was set to 10 GBaud, and the simulation bandwidth to 320 GHz. The FFE $F(z)$ was placed either at the transmitter or at the receiver side, depending on the desired configuration for each simulation. A raised-cosine-in-time-domain pulse shaping filter was used in order to convert the digital data stream into an analog signal, i.e. to perform a digital-to-analog conversion (DAC). Using an ideal laser operating at a wavelength λ_c = 1550 nm and an optical I-Q modulator, the analog signal was up-converted into the optical band-pass domain. The Mach-Zehnder modulators (MZMs) in the I-Q modulator were assumed to operate in the linear regime and therefore their nonlinear cosine characteristic was neglected. The optical signal was transmitted through a transmission link of one to 10 spans, where each span is composed of a *linear* SSMF with an attenuation coefficient α_{dB} = 0.2 dB/km and a dispersion parameter D = 17 ps/nm/km, followed by an EDFA with a noise figure (FN) of 5 dB. The received signal was then detected by an ideal homodyne coherent receiver. In the electrical domain, a fifth order Butterworth low-pass filter was applied to suppress out-of-band noise. The signal was then down-sampled to the symbol rate with an analog-to-digital converter (ADC) to yield r_k. The sampled received signal was then processed either by the DFE, or by an FFE. In case of THP, a modulo operation

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was used to force the equalized symbols to the interval [M,M) to yield y_k. For the assessment of performance, the error-vector magnitude (EVM) figure of merit was used. Note that in case of DFE it is clear that the residual error sequence is considered for the calculation of the EVM, and not its output.
For all simulation results, an EVM value equivalent to a BER of 10^{-3} is given as a reference and is marked as a dashed curve in all figures.

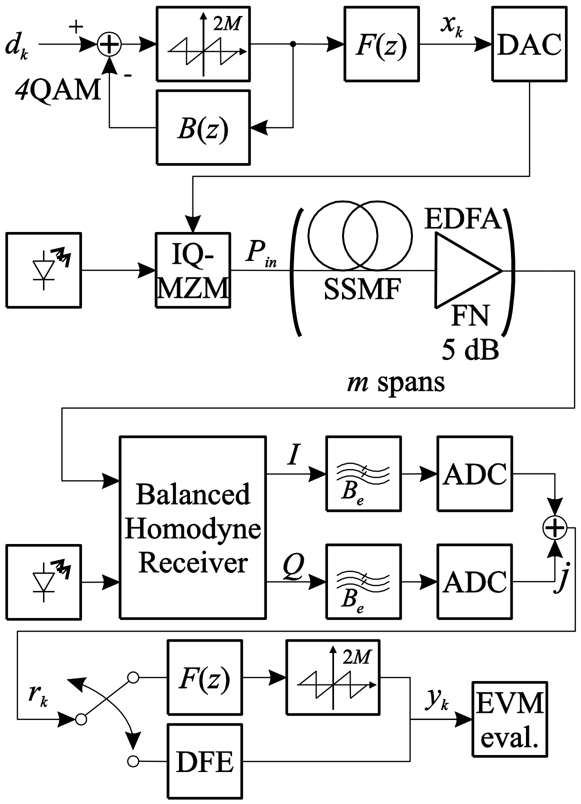

Figure 6 Simulation setup.

3.2 Receiver Filter Effects on the Performance of THP

The receiver filter is generally applied in order to suppress the out-of-band noise. In the filtering process, side lobes of the signal spectrum are filtered as well. This could serve the compensators to achieve better performance, since the unavoidable effect of aliasing is reduced. On the other hand, a narrow receiver filter alters the total impulse response of the channel, what can affect THP's extended signal set. In the following, a 4QAM signal was precoded with THP and than transmitted over 500 km, simulating a possible metro network scenario. The numbers of filter coefficients N_f and N_b were set each to four, six and eight without loss of generality. The cut-off frequency of the receiver filter was varied between $0.5R_s$ and $1.2R_s$. The results are summarized in **Figure 7**. As can be seen, the performance of THP degrades as the bandwidth of the filter becomes larger. This is clearly the result of the aliasing effect. On the other side, the extended signal set seems to grow with the reduction of the filter's bandwidth. Therefore, there is a trade-off between performance and receiver complexity by means the ADC and DSP unit resolution.

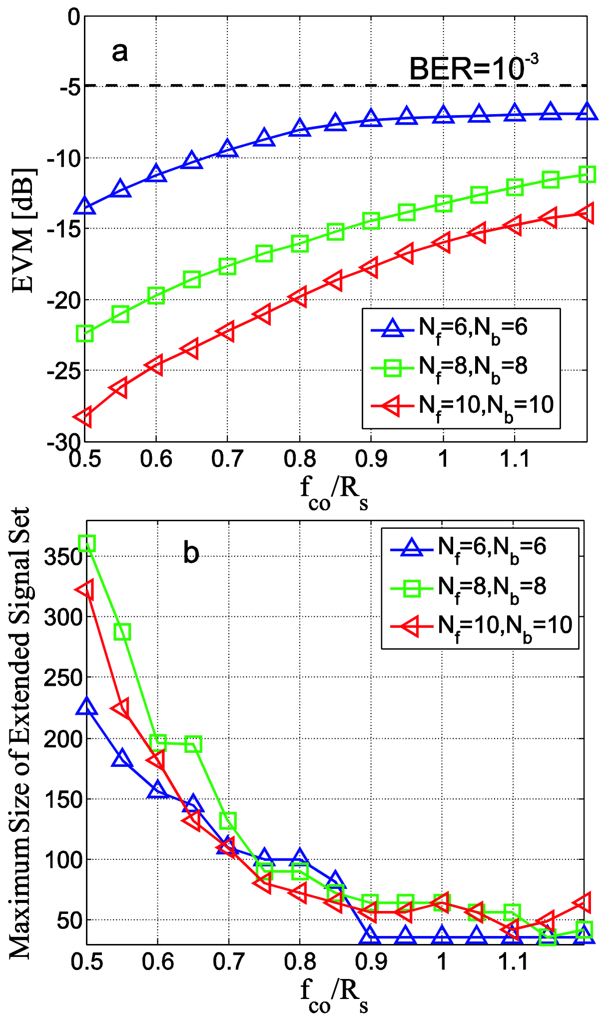

Figure 7 Effect of the receiver filter on the performance of THP: (a) EVM and (b) maximum number of symbols in the extended signal set vs. the normalized cut-off frequency of the receiver filter.

3.3 THP Performance for a Varying Number of Filter Coefficients

Assuming a sufficiently high-resolution ADC and DSP unit, the receiver filter's cut-off frequency was set to $0.5R_s$. A 4QAM signal was again transmitted over 500 km. The signal was either precoded with THP or equalized with DFE, where N_f and N_b were varied between 1 and 15. Noise was neglected to simulate a high optical SNR (OSNR) scenario. The results are shown in **Figure 8**. As expected, the performance of both compensators shows an improvement with the increasing of the number of filter coefficients, or equivalently their computational complexity. In addition, given a constraint for the total number of filter coefficients, this map of values, given in **Figure 8.a**, helps the designer of such compensators to

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decide for the optimal N_f and N_b. The results of such a design are summarized in **Figure 8.b**. By comparing the two methods, a slight better performance is registered for the DFE structure.

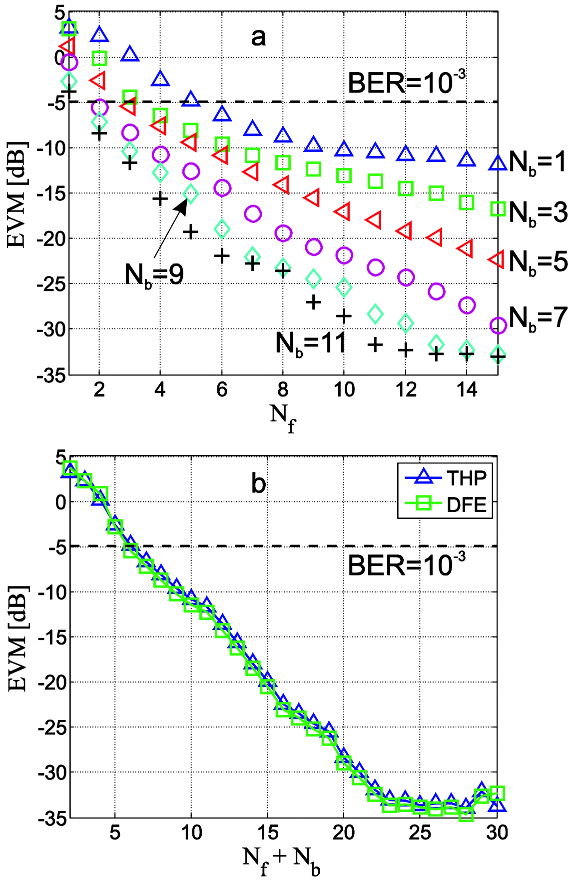

Figure 8 Compensators performance for a varying number of filter coefficients: (a) EVM vs. N_f for six different N_b values in case of THP (b) optimal EVM for a restricted total number of filter coefficients $N_f + N_b$.

3.4 THP Performance for a Varying Transmission Length

Assuming a resolution-limited ADC and DSP unit, the receiver filter's cut-off frequency was set to $1.2R_s$. A 4QAM signal was transmitted over one to 10 spans, and again was either precoded with THP or equalized with DFE. The performance of the compensators for a varying transmission length was tested for three different designs under the constraint $N_f + N_b = 14$. As in the previous simulation, noise was neglected. The results are shown in **Figure 9**. As expected, the performance of both compensators degrades when increasing the transmission length, since the length of the impulse response increases as well. In addition, it can be seen that (under the given constraint) the design with a greater N_f performs better for transmission lengths greater than 600 km. For $L_{SMF} < 600$, however, the previous argument is no longer valid. Therefore, the choice of a pair (N_f , N_b) has to be always optimized for a specific channel configuration.

By comparing the two methods, a slight better performance is registered for the DFE structure for almost all scenarios. For L_{SMF} = 1000 km and the pair (6, 8), however, the DFE structure performs significantly worse than THP. This can be attributed to the error-propagation phenomenon of the DFE, since for that design the feedback filter has more influence on the compensation performance then the feed-forward filter.

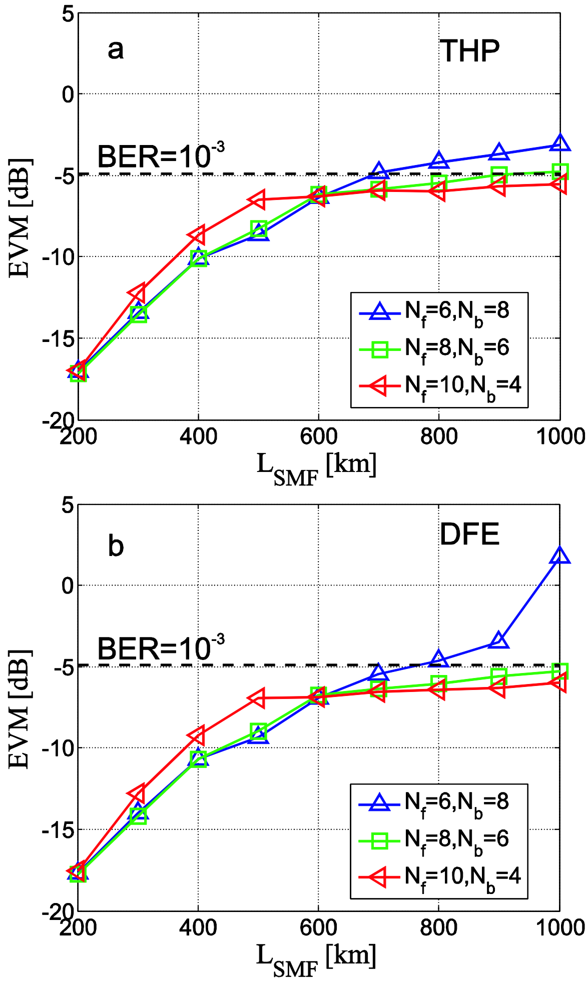

Figure 9 Compensators performance for a varying transmission length: EVM vs. the length of the transmission link in case of (a) THP and (b) DFE.

4 Possible THP Scenarios for a fiber-optic system: Discussion

As was mentioned in the introductory section, THP offers the possibility to use an equalizer with a feedback structure in conjunction with channel coding schemes such as TCM and LDPC. Another possible advantage of THP is the reduction of the receiver's digital signal processing complexity by performing it partially (or entirely) at the transmitter side. Possible scenarios for using THP in a fi-

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ber-optic transmission system should clearly exploit these advantages. On its down side, conventional implementation of THP, as shown in this paper, has to be detected coherently, since the extended signal set at the receiver side is most likely to contain complex values resulting from the fiber's complex impulse response. In fact, even when neglecting the imaginary values, the received signal will be extended over the real axis. For ASK modulation orders greater than two, applying direct detection would reduce the amount of symbols that can be detected by half, i.e. a loss of one bit per symbol. A way to overcome this challenge might be an up-conversion of the complex baseband precoded signal to a real band-pass *electrical* signal using an electrical I-Q modulator. Using a guard interval, in a similar way as been done in direct-detection optical OFDM, the signal can be detected without loss despite the use of a single photo-diode. This suggestion was not tested yet, and its compatibility is to be investigated by the author.

Under this constraint, the following scenarios are suggested: THP can be applied in optical networks, where one main node or a switch serves many end units, which operate with coherent detection.

A scenario or a transmission scheme that would be compatible with THP is polarization multiplexing (PM). This system can be modelled as a 2×2 MIMO channel. Since PM has to be used in conjunction with coherent detection, and since THP is already compatible with MIMO transmission, it is possible to replace the "butterfly" equalizer structure with a precoder.

In addition, there exist possible scenarios, where it is desired to reduce the DSP load, but a DFE structure cannot be used for that purpose. One example is no-guard-interval OFDM, where all subcarriers are received simultaneously and cannot be separated without applying equalization first. After separation, a second stage of equalization has to be performed in order to equalize for the residual ISI. These two stages can be combined into a single precoder.

5 Conclusion

In this paper, the compatibility of THP with fiber-optic communication systems was investigated. The principles of THP for fiber-optic communication system were explained, as well as its implementation requirements. The performance of THP as well as the size of the extended signal set was shown to be dependent on the receiver filter bandwidth, where a trade-off exists between them. THP performance was found to be comparable to that of DFE for metro network scenarios with high OSNR. It was shown that due to its extended signal set property and the fiber channel complex characteristic, THP has to be used in conjunction with coherent detection in order to allow the recovery of the data signal. In addition, different scenarios that can profit from the advantages of THP have been suggested. These have to be further investigated, in order to prove or revoke THP's compatibility with fiber-optic communication systems.

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