Tomlinson-Harashima Precoding for Fiber-Optic Communication Systems

Roi Rath and Werner Rosenkranz
Chair for Communications, Christian-Albrechts-Universität zu Kiel, Kaiserstraße 2, D-24143 Kiel, Germany, ror@tf.uni-kiel.de

Abstract *The performance of Tomlinson-Harashima precoding (THP) is investigated for a 12.5 GBaud 16QAM transmission. THP is shown to reduce significantly the computational load compared with linear equalization and to eliminate the error propagation phenomenon associated with decision-feedback equalization.*

Introduction

In order to address the equalization problem in a digital system, zero-forcing equalization (ZFE) with a decision-feedback equalizer (DFE) can be used in order to achieve exact equalization for sampling at the Baud rate[1]. Under the assumption of correct decisions, the DFE removes completely the ISI and leaves the white noise uncolored. In addition, it was shown that DFE, in combination with powerful channel coding, allows transmission to approach the channel capacity[2]. On its down-side, DFE has two main disadvantages, namely the so-called *error propagation* effect, and the inability to *easily* incorporate itself with channel coding techniques, such as trellis-coded modulation (TCM) or low-density parity code (LDPC)[2].

Tomlinson-Harashima precoding (THP)[3,4] is a pre-compensation method, which offers an alternative to the DFE with the possibility to apply channel coding *conjunctively*[2]. This is achieved by moving the DFE's feedback structure to the transmitter side, where the decision device is replaced with a zero-mean modulo operation, as illustrated in Fig. 1. THP is very popular in the field of wireless communications, namely in multiple-input multiple-output (MIMO) channel scenarios.

Due to the progressive research to develop faster electronic circuits[5] and the availability of coherent detection, many compensation algorithms are adopted from the field of digital communications to be used in fiber-optic communication systems. THP, however, and to the best of our knowledge, hasn't yet been proposed for pre-compensation of fiber-optic transmission impairments. In this paper, the compatibility of THP with fiber-optic communication systems is investigated.

THP for Fiber-Optic Communication Systems

Similarly to DFE, the THP structure shown in Fig. 1 can be applied efficiently for causal channels, i.e. for channels with postcursor ISI only[1]. The optical communication channel,

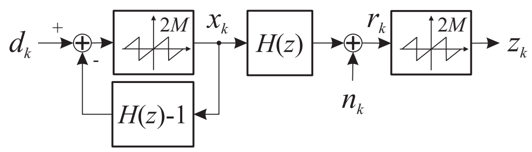

Fig. 1: A communication system with THP. Here, the data, precoded, white Gaussian noise, received and the decoded sequences are denoted respectively by d_k, x_k, n_k, r_k, and z_k. In addition, $H(z)$ is the channel transfer function.

however, has in fact a non-causal complex impulse response with both postcursor and precursor ISI components. Therefore, it is necessary to use a feed-forward equalizer (FFE) in conjunction with THP. Since the transfer function of the discrete impulse response of the channel may contain zeros close to the unit circle, the FFE $F(z)$ is chosen to be positioned at the receiver side in order to avoid a possible transmit signal power enhancement. For the derivation of the optimum filter coefficients, it is possible to use the well-known minimum mean square error (MMSE) algorithm[6].

The *precoded* sequence x_k is characterized as almost white and uniformly distributed in the square $[-M-jM, +M-jM, -M+jM, +M+jM]$, where M is the number of signal levels of the real or imaginary component of a square QAM modulation. A side-effect of the precoding is a slightly higher transmit power, which is however negligible for higher order modulation. At the receiver side, the equalized signal before decoding or the *effective data sequence*[2] can be defined as (neglecting noise):

$$y_k = d_k + p_k \,, \qquad (1)$$

where p_k is the *precoding* sequence and can get the values $2Ma+j2Mb$, $a, b \in \mathbb{Z}$. Using the same modulo operator as used at the transmitter, the contribution p_k is cancelled and the original data sequence is retrieved. The dynamic range of the effective data sequence is both M and channel

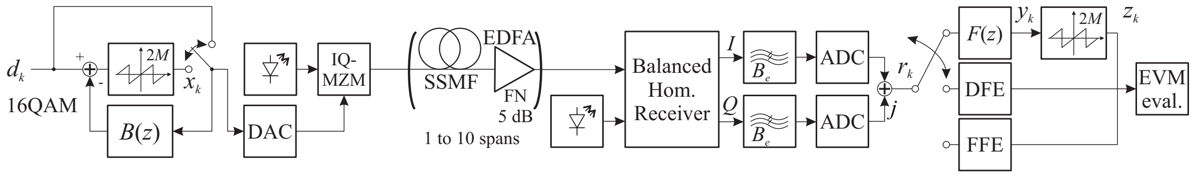

Fig. 2: A block diagram of the investigated system. Here, d_k and x_k are the data and precoded sequences, respectively, r_k is the coherently received and sampled signal, and y_k and z_k are the equalized and decoded sequences, respectively. In addition, $F(z)$, $B(z)$ and $H(z)$ are the feed-forward, feedback and channel transfer functions, respectively. The cut-off frequency of the receiver filter is denoted as B_e.

energy dependent. Therefore, it can become very large, what requires a higher resolution DSP unit.

In addition to standard implementation requirements in coherent optical systems, THP requires a modulo function both at the transmitter and receiver, which can be easily performed using the two's-complement representation in the digital circuit[3]. In addition, the channel information has to be known at the transmitter side. It is also possible to apply THP even if only reduced channel state information is available at the transmitter[7].

System Setup

A schematic of the system used for the investigation is given in Fig. 2. A 2^{16} long De-Bruijn pseudo random hexadecimal sequence was mapped to 16QAM symbols d_k, which were processed with THP to yield the precoded symbols x_k. A symbol rate of 12.5 GBaud plus 7% of FEC overhead (denoted as R_s) was chosen, in order to achieve a net data rate of 50 Gbits/sec. Digital-to-analog conversion (DAC) was achieved by filtering the precoded sequence with a raised-cosine-in-time-domain pulse shaping filter. Using an ideal laser operating at a wavelength λ_c = 1550 nm and an optical IQ modulator, the analog signal was up-converted into the optical band-pass domain. The Mach-Zehnder modulators (MZMs) in the IQ modulator were assumed to operate in the linear regime. The optical signal was transmitted through a transmission link of one to 10 spans. Each span was composed of a *linear* SSMF with an attenuation coefficient α_{dB} = 0.2 dB/km and a dispersion parameter D = 17 ps/nm/km, followed by an EDFA with a noise figure (FN) of 5 dB. The received signal was detected by a homodyne coherent receiver, followed by a fifth-order *Butterworth* filter with a one-sided bandwidth B_e, and sampled with an analog-to-digital converter (ADC) operating at the symbol rate to yield r_k. The sampled received signal was processed either with $F(z)$ (associated with THP), a DFE or an FFE. In case of THP, a modulo operation was used to force the equalized symbols y_k to the square [−4−j4, +4−j4, −4+j4, +4+j4] to yield z_k. For the assessment of performance, the error-vector magnitude (EVM) figure of merit was used. For all simulation results, an EVM value equivalent to a BER of 10^{-3} is given as a reference and is marked as a dashed curve in all figures. In addition, the effects of laser phase noise and polarization-mode dispersion were neglected.

Results

As a first step, the bandwidth of the receiver filter was optimized. The receiver filter, which is generally applied in order to suppress the out-of-band noise, filters the side lobes of the signal spectrum as well. This could serve the compensators to achieve better performance, since the unavoidable effect of aliasing is reduced. Fig. 3 shows the performance of THP versus the cut-off frequency, which was varied between 0.1R_s and 1R_s. The transmission length was set to 500 km, and the numbers of coefficients for the feed-forward (N_f) and feedback (N_b) filters were set each to six, eight, 10 and 12. As can be seen, the optimal cut-off frequency is around 0.3R_s. From this point the optimal receiver filter bandwidth was considered, unless otherwise mentioned.

Next, THP performance was compared with that of DFE and FFE for a varying total number of equalizer coefficients $N_{tot} = N_f + N_b$. In case of THP and DFE the choice of N_f and N_b for a

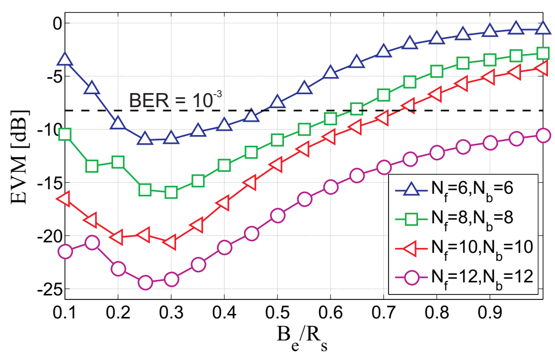

Fig. 3: THP performance vs. the bandwidth of the receiver filter for different equalizer orders (transmission length 500 km). The EVM is minimized around 0.3R_s Hz, where the effect of aliasing is minimal.

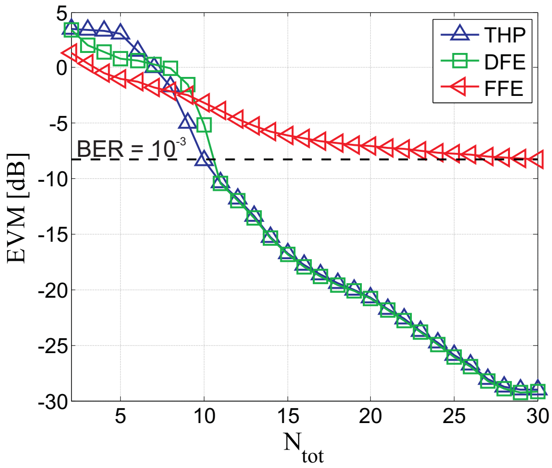

Fig. 4: Compensators performance vs. total number of equalizer coefficients $N_{tot} = N_f + N_b$, where N_f and N_b are number of feed-forward and feedback filter coefficients, respectively. For FFE $N_{tot} = N_f$.

given N_{tot} was optimized to maximize the performance. Results are given in Fig. 4. As expected, the performance of all equalizers improves with the increasing number of coefficients. For $N_{tot} \geq 11$, THP clearly outperforms MMSE-FFE, whereas in comparison with DFE the same performance is obtained. For $N_{tot} \leq 8$ THP and DFE register higher EVM values compared to FFE. Performance degradation in case of DFE is caused by error propagation, whereas for THP it is the result of mapping of received signal values, which are over the modulo threshold, to the opposite side of the constellation. This phenomenon, which is illustrated in Fig. 5, is fortunately negligible around the FEC limit, what cannot be said about the error propagation effect of the DFE.

Last, the performance of all compensators for a varying transmission length was tested for design constraints of $N_{tot} = 15$ and $B_e = 0.3R_s$. Again, the choice of N_f and N_b was optimized to

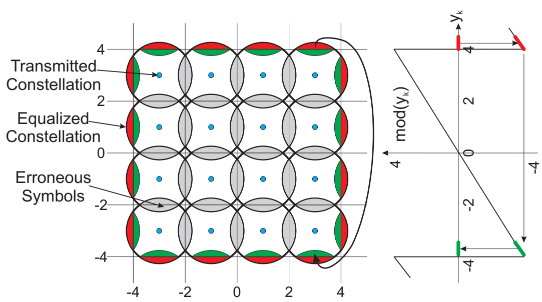

Fig. 5: THP additional error phenomenon. The equalized constellation is illustrated as circles. The grey areas contain erroneous symbols. The red areas, which will normally be detected correctly, are modulo-mapped (see modulo function right side) to the opposite side of the constellation (green area), and therefore cause additional errors.

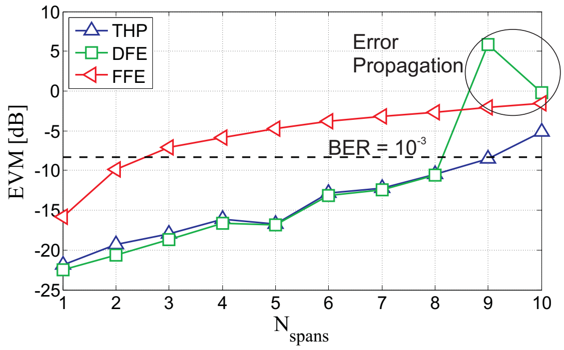

Fig. 6: Compensators performance vs. number of spans. For all compensators $N_{tot} = 15$.

maximize the performance. The results are shown in Fig. 6, and as expected, the performance of all compensators degrades when increasing the transmission length, since the length of the impulse response increases as well. Under the given constraints, FFE fails after 250 km, whereas DFE and THP equalize successfully till 800 and 900 km, respectively. Again, error-propagation is visible in case of 9 and 10 spans. The fact that THP is error propagation free allows much more flexibility in the equalizer design with respect to system constraints.

Conclusions

In this paper the compatibility of THP with fiber-optic communication systems was investigated. Simulation results have shown that the performance of the compensators under investigation is receiver bandwidth dependent, and should be considered in the equalizer design. THP was shown to outperform MMSE-FFE, and to offer a similar performance as of DFE, with the advantage of the elimination of error propagation, which was shown to be catastrophic in the vicinity of the FEC limit. In addition, THP was found to significantly increase the transmission length in comparison with FFE.

Acknowledgements

This work was supported by BMBF project SASER under contract number 16BP12404.

References

[1] K.-D. Kammeyer, *Nachrichtenübertragung*, Wiesbaden: Vieweg+Teubner Verlag (2008).
[2] R.H.Fischer, *Precoding and Signal Shaping for Digital Transmission*, New York: John Wiley & Sons, Inc (2002).
[3] M. Tomlinson, Elec. Let. **7**, 138-139 (1971).
[4] H. Harashima et al., IEEE Trans. on Comm. **20**, 774-780 (1972).
[5] I. Dedic, Proc. OFC'10, OThT6 (2010).
[6] R. A. Casas et al., DFE Tutorial, (1998).
[7] A. Liavas, IEEE Trans. on Comm. **53**, 5-9 (2005)