# Equalizer Complexity of Mode Division Multiplexed Coherent Receivers

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**Abstract:** We show that OFDM requires the lowest equalizer complexity for crosstalk compensation in a mode-division-multiplexing receiver. For a 2000-km transmission distance and less than 10% OFDM-specific overhead, the modal dispersion must be below 12 ps/km.
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## 1. Introduction

The increasing capacity demand for long-haul networks has almost exhausted the capacity limits of conventional single mode fiber (SMF). The most promising technology for capacity expansion is space division multiplexing, which exploits the fact that capacity can be multiplied by transmitting information along different paths or transmission modes. Among the various different approaches for space division multiplexing, few mode fibers (FMF) attracted a lot of attention [1]. In [3], the transmission over 40 km of FMF with three-fold mode-multiplexing has been demonstrated using an FMF with a 4.35 ps/m mode differential modal delay (DMD). More recently, there has been also progress in transmission with the aid of FMF amplification, and thereby show the possibility to further extend the maximum feasible transmission distance on FMF [4].

The most optimized design of an FMF is still an active area of research. Fibers with a high mode group delay increase the necessary number of taps for equalizers in multiple-input multiple-output processing (MIMO), which might make mode-multiplexing impractical with the DSP complexity available in the upcoming years. For example, Randel et al. [7] used 36 finite impulse response filters with 120 taps for a 6 x 6 MIMO equalizer using a three-fold FMF to compensate the modal crosstalk accumulated in 33 km of transmission.

In this paper, we analyze the complexity of various equalization schemes of coherent receivers for both blindly adapted and training symbol (TS) based approaches with respect to the equalization of modal dispersion and crosstalk in a FMF. We limit the TS and cyclic prefix (CP) overheads in total to a maximum 10% for the TS based scenario. The complexity is calculated in terms of complex multiplications per transmitted bit. There is a significantly larger difference between TS and blindly adapted schemes compared to what we observed for SMF [8]. For FMF, among TS based algorithms, orthogonal frequency division multiplexing (OFDM) provided the lowest complexity. However, due to the 10% overhead constraint, the maximum reach is limited by mode division multiplexing.

## 2. Equalizer Complexity for Two-Mode Fiber

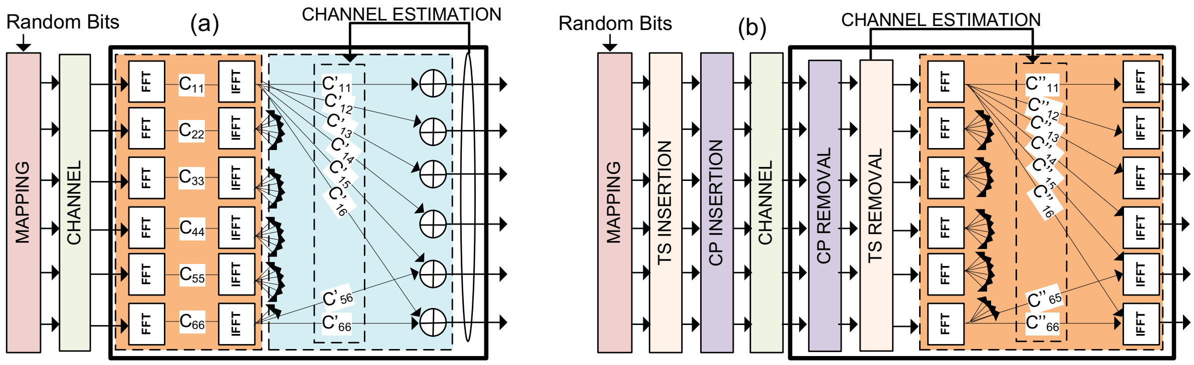


Fig. 1 Equalizers for 6 x 6 FMF with an example for (a) blind equalizer, for single carrier system with FDE/TDE and (b) TS based equalizer, single carrier system with FDE.

The equalizer schemes can be divided into two groups: blindly adapted and TS based. Fig. 1 depicts two equalizer examples from each group for FMF. In this work the most common equalizer schemes are compared in terms of complexity for FMF equalization. The time domain equalizer (TDE) and hybrid frequency domain time domain equalizer (FDE/TDE) are investigated for blindly adapted approach. FDE, OFDM and hybrid FDE/OFDM are studied for the TS based approach. The complexity is assessed in terms of complex multiplications per transmitted bit [8], for a 6x6 configuration, a two-mode FMF with three-fold mode-multiplexing ($LP_{01}$, $LP_{11a}$ and $LP_{11b}$), and polarization-multiplexing is assumed for all configurations. The complexity calculation includes forward error correction bits but excludes TS based equalization specific overhead bits. The channel is assumed to be linear, nonlinear effects and laser phase noise are not taken into account. The blind equalizer example in Fig. 1(a), single carrier FDE/TDE, converts signal into the frequency domain with FFT, compensates for the chromatic dispersion (CD), and goes back into time domain with FFT (FDE part) where the butterfly structure (TDE part) is used for differential group delay (DGD) compensation. For FMF, both DMD and polarization mode dispersion (PMD) delay contribute to DGD. Some of the branches in the butterfly structure are not shown for simplification of the figure. The coefficients of the CD compensation block are estimated only once at the beginning of transmission. The CD compensation part contributes FFT complexities along with the complex multiplications required with coefficients ($C_{11}$, …, $C_{66}$). The DGD compensation coefficients are time dependent and updated blindly once per symbol duration. The TDE part contributes of complex multiplications with coefficients ($C'_{11}$,…, $C'_{66}$) as well as the coefficient update. The TS based example is single carrier FDE. The cyclic prefix (CP) and TS are inserted at transmitter side, and both add an overhead to the total data rate. The TSs are then used to calculate equalizer coefficients at the receiver. These coefficients are updated with a 10 μs repetition rate. The CP is long enough to accommodate the impact of CD and DMD, and is subsequently removed at the receiver. For OFDM, IFFT is done at transmitter side rather than at the receiver. The complexity calculation for one frame includes the FFT complexity, calculation of channel matrices and their inverse at every channel estimation update, and the multiplication of the received frame with the inverse channel matrix.

## 3. Analytical Results

The net bit rate per mode is 100 Gb/s using QPSK. The FEC overhead is chosen as 11% and the combined maximum overhead for TS and cyclic prefix (CP) is 10% for TS based approach. The oversampling for both single and multi carrier schemes are 1.5. The FMF has two modes with three-fold multiplexing; $LP_{01}$, $LP_{11a}$ and $LP_{11b}$, and each polarization-multiplexed. The approximation for the time shift between tributaries, due to both the DMD and PMD, is denoted by

$$\tau = p \cdot \sqrt{d} + \chi \cdot d \ , \qquad (1)$$

where $p$ is the PMD parameter, 0.02 ps/√km, $d$ is the distance in km, $\chi$ is the DMD per km, i.e. modal dispersion, in ps/km. The chromatic dispersion parameter is chosen as 20 ps/nm/km to be compatible with recent publications such as [6].

The TS based equalization technique uses one TS for FMF, which is valid under the assumption that with subcarrier multiplexing one can find orthogonal symbols [9]. Consequently, one TS is sufficient for the 6 x 6 scenario and the TS overhead is kept as low as possible. The channel estimation is updated every 10 μs so that TSs can dynamically react to changes in optical channel [10], however, the optimum repetition rate for FMF should still be investigated. The complexity is calculated in a two-dimensional matrix, one variable is the sampling rate and the other is the size of the FFT. For each configuration, we find the optimum sampling rate resulting in minimum complexity satisfying the overhead constraint.

As a reference distance we assume here transmission over 2000-km of FMF. Fig. 2(a) shows the complex multiplications per bit for the equalizer types described above as a function of modal dispersion of FMF for 2000 km. The blind equalizers, TDE and FDE/TDE, use more than 1000 multiplications even for a modal dispersion of 5 ps/km, which is highly impractical. Most of the multiplications required for FDE/TDE are due to time domain operations. The TS based equalizer complexity is significantly lower; however the reach is limited by the maximum allowed overhead. Among these, OFDM has the lowest complexity. However, it cannot tolerate more than a modal dispersion of 12 ps/km due to the overhead constraint. At this point, the number of complex multiplications per bit for OFDM is only 13.7.

The best TS based (OFDM) and blind equalization (FDE/TDE) techniques are chosen for Fig. 2(b). In this figure, the equalizer complexity is shown versus distance, and we directly compare the required complexity for distortion equalization of both SMF and FMF. The modal dispersion for FMF is chosen as 27 ps/km since this is the lowest value seen in literature up to now to our knowledge [5]. The length of impulse response in multiples of sampling duration ($N_{IR}$) for FMF increases up to 2276 at 2000 km for FMF whereas it does not exceed 1 for SMF which is only due to PMD. The $N_{IR}$ calculation can be found in [8]. The FDE/TDE equalizer complexity is highly dependent on the time domain operations which are determined by τ. The large difference in τ between SMF (2 x 2) and FMF results in the much higher FDE/TDE complexity for FMF, whereas the complexity of FDE/TDE for SMF is only 12.7 multiplications per bit at 2000 km. The time shift, τ, is compensated with the CP for the TS based equalizers, and as a result the FFT is dominating complexity. The number of required FFTs scales with the number of modes. For instance 4 FFTs are required for SMF and 12 for FMF. However, the number of output bits of the equalizer increases proportionally with the number of modes as well. Consequently, the size of the FFT determines the complexity per bit rather than the number of FFTs used. As a result the complexities of a TS based equalizer for FMF are not significantly different than for SMF. On the other hand, the equalization scheme chosen for this figure, OFDM, reach cannot exceed 689 km again due to the 10% overhead constraint. The FDE/OFDM approach can tolerate 23.8 ps/km modal dispersion with higher number of complex multiplications, i.e. 32.3.

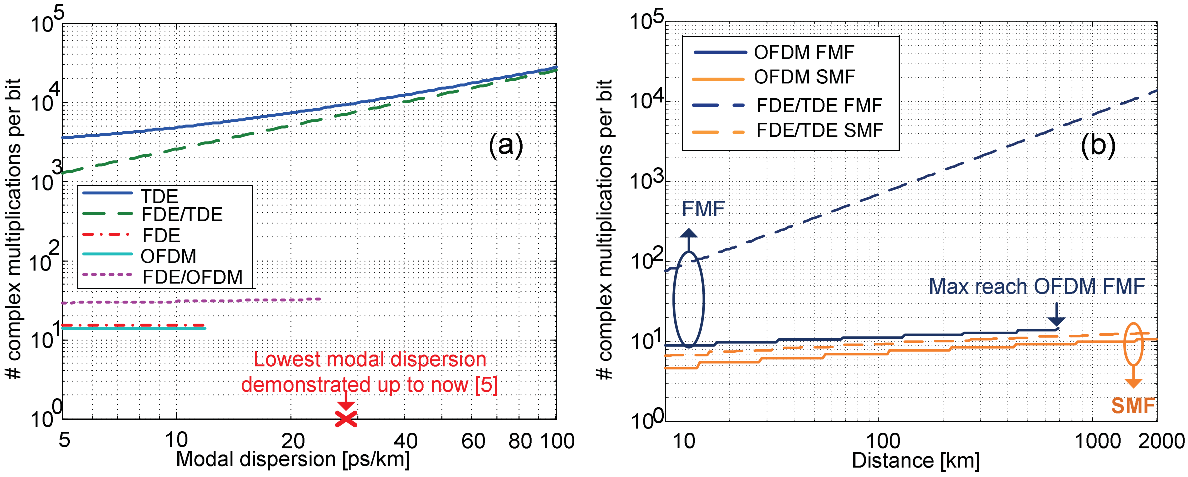


Fig. 2 Complex multiplications per bit in terms of modal dispersion for 2000 km. Total overhead for cyclic prefix and training symbols is maximum 10%. (b) Complex multiplications per bit in terms of distance for few mode and single mode fiber. The PMD is 0.02 ps/√km for both cases and the modal dispersion is 27 ps/km for FMF.

## 4. Conclusion

In this paper, we showed that the complexity of blind equalization based DSP approaches for FMF is not likely to be practical for many years to come when long-haul transmission distances are targeted. A training symbol based approach can lower the DSP complexity with several orders of magnitude, but requires additional overheads. Limiting these additional overheads to 10%, a fiber with a modal dispersion as low as 12 ps/km is required for 2000 km of multi-mode transmission.

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