# OSNR Sensitivity of Multi-Level Modulation Formats

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**Abstract:** A simple analytical method to estimate the OSNR sensitivity of multi-level amplitude, phase and combined modulation formats is shown. Results are verified by numerical simulations.

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## 1. Introduction

The trend to higher data rates in optical transmission systems and the requirement of higher spectral efficiency has led to the use of optical multi-level modulation formats, where multiple information bits are encoded in a single symbol [1]. While the spectral width of the signal is reduced and the spectral efficiency in WDM systems is increased, the signal becomes more sensitive to noise added in optical amplifiers along the transmission line and therefore is limited in system reach. Thus, a comparison of different modulation formats for a given per-channel data rate must take into account the achievable capacity of a transmission system as well as the OSNR tolerance. Here, we propose a method to get a quick overview of the OSNR tolerance of various multi-level amplitude and phase modulation formats and verify these approximate results against more detailed simulation results.

## 2. Approximation Method and Simulations

The electrical field vector of an amplitude and/or phase modulated optical signal can be depicted in a constellation diagram in the complex plane, as shown in Figure 1 for the example of 16-QAM. By using $M$ constellation points, $b$=ld($M$) bits can be encoded into one symbol. The value $b$ can be a non-integer number [2]. During propagation in the transmission system, optical noise is added to the signal by optical amplifiers. Here, we assume white noise with a Gaussian amplitude distribution and uniformly distributed phase (noise amplitude $N$ in Fig. 1).

The minimum distance $D$ to the next constellation neighbor (see Fig. 1) determines the sensitivity of the modulation format towards the noise impairment. We assume a linear detection, like in a coherent receiver. However, the approximate results can also be applied to direct detection receivers. The optical signal to noise ratio (OSNR) is the ratio of the average signal power $P_{avg}$ (average of squared normalized constellation vectors) and the optical noise power in a fixed optical bandwidth:

$$OSNR = \frac{P_{avg}}{N_{opt}^2} \quad (1) \qquad SNR = \frac{D^2}{N^2 \cdot b_{rel}} \quad (2) \qquad OSNR_{Penalty} = \frac{4 \cdot P_{avg} \cdot b_{rel}}{D^2} \quad (3)$$

The noise present at the decision circuit is dominated by the filtered optical noise and therefore for higher-order modulation formats reduced by a bandwidth factor $b_{rel}$ compared to binary signaling. The signal to noise ratio (SNR) after filtering is approximated as (2). To compare different modulation formats, the required OSNR is related to a fixed SNR, determined by the constellation distance $D$. If we normalize to the required OSNR for the 2-PSK format, we find a penalty relative to 2-PSK of an arbitrary multi-level format by the simple approximation (3).

For verification, numerical Monte Carlo simulations are performed. Gray-coded M-ary symbols with 50% return-to-zero (RZ) pulse shaping are transmitted using coherent homodyne detection. Ideal synchronization of the carrier laser and the local oscillator at the receiver and negligible laser linewidths are assumed. Optical and electrical filter bandwidths at RX (3$^{rd}$ order Gaussian, optical Bessel 5$^{th}$ order electrical) are optimized. The OSNR penalty compared to 2-PSK is calculated for BERs of $10^{-3}$ and $10^{-5}$.

## 3. Results

In Table 1, for various multi-level modulation formats the values of normalized average power ($P_{avg}$), normalized constellation point distance ($D$) and relative bandwidth ($b_{rel}$) are summarized and the resulting OSNR penalty is evaluated using Eq. (3). In addition, the general case of $M$ modulation levels is shown. Based on these equations, the OSNR penalties are shown in Fig. 2. It is found that optimum OSNR tolerance can be obtained for a 3-PSK modulation, where the required OSNR is approximately 0.8 dB lower than for 2-PSK or 4-PSK. Using this modulation format, >1.5 bit can be encoded into one symbol, or, 3 bits can be encoded in two symbols.

The last row shows the simulated OSNR penalty. The results for a BER of $10^{-5}$ are included in Fig. 2 as circles. Only minor deviations from the theoretical values are visible. For 3-PSK, 3 bits are encoded into 2 symbols, which reduces the theoretical achievable OSNR improvement over 2-PSK to 0.5 dB. In case of unipolar ASK it can be seen that for higher order modulation the simulated OSNR values differ more from the theoretical ones than in case of e.g. PSK. This results from the severe influence of the filtering in optical and electrical domains (no matched filters). A trade-off between noise reduction and introduction of inter symbol interference (ISI) due to narrow filter bandwidth has to be made. The results for BER of $10^{-3}$ and $10^{-5}$ differ for higher-order ASK, because the sensitivity towards ISI leads to a shallower slope in the BER vs. OSNR curve. A minor inaccuracy of the approximation method is also based on the fact that some constellation points have more “nearest neighbors” than others (compare for instance, 2-PSK to inner constellation points of 16-QAM, Fig. 1). This difference, however, results in a minor OSNR deviation and is partly mitigated by using error-tolerant Gray coding[3].

## 4. Discussion

So far, we have assumed a single signal polarization. For this, the required OSNR is the same as for a polarization multiplex signal with half the symbol rate. We considered coherent detection, but applicability on direct detection is indicated by additional simulations and by [4]. Since we only considered the OSNR tolerance of the modulation schemes, we did not discuss non-linear effects in the transmission fiber like self-phase modulation, which can have a higher impact on some of the schemes due to amplitude dependent phase modulation. This is discussed e. g. in [5].

## 5. Conclusion

We proposed a simple method to estimate the OSNR penalty of a variety of amplitude and phase modulation formats. The analytical results are in good agreement with the numerical simulation results. Minor differences are due to inter-symbol interferences, which tend to be more severe for higher-level ASK and which are not included in the simple estimation. The most noise tolerant format turned out to be a 3-level phase shift keying (3-PSK).

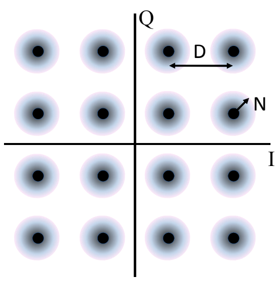


Fig. 1: Representation of constellation points in the complex (I-Q) plane for the example of 16-QAM. D: distance between neighbor constellation points, N: measure for the noise amplitude.

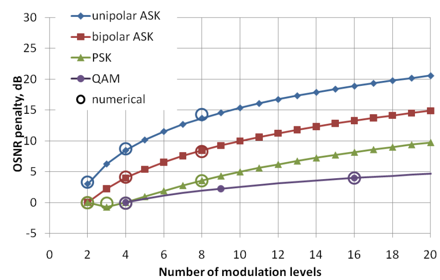


Fig. 2: OSNR penalty of multi-level formats. Diamonds: uni-polar ASK, squares: bi-polar ASK, triangles: PSK, dots: square QAM. Circles: numerical OSNR penalty for BER $10^{-5}$ (normalized).

| | Unipolar ASK | | | | Bipolar ASK | | | | PSK | | | | | Square QAM | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # levels | 2 | 4 | 8 | M | 2 | 4 | 8 | M | 2 | 3 | 4 | 8 | M | 4 | 16 | M |
| $P_{avg}$ | 0.5 | 0.389 | 0.357 | $\frac{(2M-1)}{6\cdot(M-1)}$ | 1 | 0.556 | 0.429 | $\frac{(M+1)}{3\cdot(M-1)}$ | 1 | 1 | 1 | 1 | 1 | 2 | 1.111 | $\frac{2\cdot(M-1)}{3\cdot(\sqrt{M}-1)^2}$ |
| D | 1 | 0.333 | 0.143 | $\frac{1}{(M-1)}$ | 2 | 0.667 | 0.286 | $\frac{2}{(M-1)}$ | 2 | 1.732 | 1.414 | 0.765 | $2\cdot\sin\left(\frac{\pi}{M}\right)$ | 2 | 0.667 | $\frac{2}{(\sqrt{M}-1)}$ |
| $b_{rel}$ | 1 | 1/2 | 1/3 | $\frac{1}{ld(M)}$ | 1 | 1/2 | 1/3 | $\frac{1}{ld(M)}$ | 1 | 0.631 | 1/2 | 1/3 | $\frac{1}{ld(M)}$ | 1/2 | 1/4 | $\frac{1}{ld(M)}$ |
| OSNR penalty | 2 | 7 | 23.33 | $\frac{2(2M-1)(M-1)}{3\cdot ld(M)}$ | 1 | 2.5 | 7 | $\frac{(M^2-1)}{3\cdot ld(M)}$ | 1 | 0.841 | 1 | 2.276 | $\frac{1}{\sin^2\left(\frac{\pi}{M}\right)\cdot ld(M)}$ | 1 | 2.5 | $\frac{2\cdot(M-1)}{3\cdot ld(M)}$ |
| OSNR penalty, dB | 3 | 8.5 | 13.7 | *See Fig. 2* | 0 | 4 | 8.5 | *See Fig. 2* | 0 | -0.8 | 0 | 3.6 | *See Fig. 2* | 0 | 4 | *See Fig. 2* |
| Simulation OSNR penalty, dB @BER $10^{-3}/10^{-5}$ | 3.2 /3.3 | 8.4 /8.7 | 13.7 /14.3 | | 0 /0 | 3.8 /4.1 | 8.2 /8.3 | | 0 /0 | 0 /-0.1 | 0 /-0.1 | 3.2 /3.5 | | 0 /-0.1 | 3.8 /4.0 | |

Table 1: OSNR penalty for multi-level formats. Constellations of bipolar 2-ASK / 2-PSK and 4-PSK / 4-QAM are equivalent, respectively.

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