Simple Improvement of Error Correction Capability of Standard FEC using Repeated Trial-and-Error Method

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Abstract: Error correction capability of standard RS(255,239)-code is optimized within the limits of hard-decision decoding. An improvement of about 0.2 dB is achieved without noticeable increase in decoding complexity.

OCIS codes: (060.2330) Fiber optics communications; (060.2360) Fiber optics links and subsystems

1. Introduction

In 2004, a new recommendation for FEC functions having higher-correction capability than the well known standard RS(255,239)-Code was approved [1]. The new standard is characterized by coding gain in the range of 7-8 dB compared to 5.6 dB for standard FEC (BER=10^{-12}).

Nevertheless, improvement of decoder performance for RS-code using techniques like soft decision and iterative decoding is still a current topic [2]. Unfortunately, despite of higher effort the increase of coding gain is low and takes values in the range of 0.5 to 1.5 dB for long codes like RS(255,239). The intention of this paper is to exhaust error correction capability of standard FEC while restricting to hard-decision decoding (HDD) without making use of reliability information. Although the improvement of 0.2 dB is low compared to what can be achieved by super FEC, it is shown that in contrast to the majority of the super FEC algorithms the additional effort is negligible, so basically the improvement comes for free. Furthermore, a number of concatenated FEC functions [1,3] make use of RS coding so that further improvement of those FEC functions is achievable using the proposed method.

The basic idea of the algorithm is depicted in sections 2 and 3. Sections 4 and 5 show expected improvement of coding gain and additional effort introduced by the algorithm, confirmed by Monte-Carlo simulations in section 6.

2. Background

Within this paper all considerations are focused on the special but important case of RS(255,239)-code characterized by length of code word n=255, message length k=239, minimum distance $d_{min}=n-k+1=17$ resulting in error correction capability $t=(d_{min}-1)/2=8$ and a number of m=8 bit for one symbol.

After encoding, each block consists of $m{\cdot}n$=2040 bit which can also be interpreted as a word of 255 symbols having m=8 bit per symbol. As the error correction capability is equal to t=8, after transmission the correct codeword can be recovered from the received word provided that not more than 8 symbols are affected by errors. As a consequence, RS(255,239) is able to correct at least 8 bit errors (in case all errors affect different symbols) and up to 64 bit errors (in case all these errors are restricted to 8 symbols). In a realistic scenario of uncorrelated errors, however, the majority of symbol errors is caused by a single bit error.

The RS-code is a non-perfect code [4]. This essential property is explained by means of the following quantitative consideration: For RS(255,239), $2^{mn}=256^{255}$ possible received words are existing. The number of valid code words is given by the number of possible message words and is equal to $2^{mk}=256^{239}$. Consequently, the number of possible received words is larger than the number of valid code words by a factor of $256^{255-239}\approx 3.4\cdot 10^{38}$. The number of correctable received words per valid codeword can be computed by determining the number of received words that differ from a specific valid codeword by not more than t=8 symbols and is given by

$$\sum_{r=1}^{t}\binom{n}{r}\cdot\left(2^m-1\right)^r=\sum_{r=1}^{8}\binom{255}{r}\cdot 255^r\approx\binom{255}{8}\cdot 255^8\approx 7.1\cdot 10^{33}. \tag{1}$$

Thus, for any valid codeword there are $7.1\cdot 10^{33}$ words assigned to this specific codeword. As the minimum distance is equal to d_{min}= 17, all correctable received words assigned to one specific codeword are different from those assigned to any other valid codeword, i.e. there is no ambiguity. Thus, the total number of correctable received words is given by $256^{239}\cdot 7.1\cdot 10^{33}$. For a perfect code, this number is equal to the total number of possible received words, i.e. any received word is assigned to one certain valid codeword. For the RS-code, the ratio of possible received words and the total number of correctable received words is equal to $256^{255}/(256^{239}\cdot 7.1\cdot 10^{33})\approx 50000$. This means that only 0.002% of all received words result in successful decoding while all others yield decoder failure.

3. Basic Idea and Extended Decoding Algorithm

The design procedure of the RS-code guarantees that any pair of valid code words differs at least by the minimum distance. On the other hand, due the large number of 'wasted' received words yielding decoder failure, the conclusion is drawn that the average Hamming distance between 'adjacent' valid code words is larger than d_{min}. Therefore, the majority of all possible received words is wasted by the algebraic hard-decision decoding procedure instead of trying to assign them to the 'nearest' valid codeword, i.e. a codeword with minimum Hamming distance.

Several approaches achieve improvement in coding gain by using reliability information, which requires costly quantization of the received signal instead of a simple decision device. In contrast, the approach proposed in this paper achieves improved coding gain within the limits of HDD. The idea is explained as follows:

We assume a received word that differs from its original codeword by t+1=9 symbols. The probability that at least one of those symbol errors is due to a single bit error is close to 100%. Now, the first one of the 2040 received bits is inverted, followed by decoding procedure. For the case of decoder failure the previously inverted bit is restored (see figure 1) followed by the same procedure with the next bit (and so on). After a certain number of iterations a single bit error is corrected and the decoder is able to correct the remaining t=8 symbol errors.

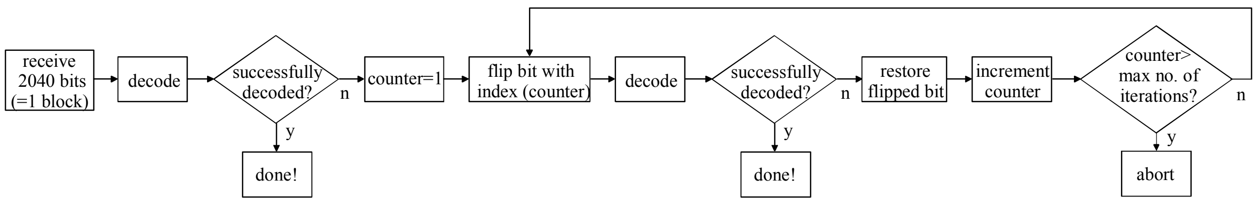

Figure 1: Extended decoding algorithm for proposed method.

As long as the number of symbol errors is less than or equal to t+1=9 and at least one single error occurs, this algorithm yields successful decoding of t+1 errors except for the case that there is another valid codeword having the same distance of t+1. In the latter case there is a certain probability that erroneously the decoding procedure results in a wrong codeword. In case there are more than 9 errors continuously decoding fails and the algorithm is aborted when reaching the maximum number of iterations (see also section 5).

Obviously, this method is optimum for HDD without using reliability information: Substituting t by t+1 in (1), the ratio of 'wasted' received words to 'used' received words is reduced from ≈50000 to ≈15. Correcting t+2 errors following the same strategy is not possible as by substituting t by t+2 in (1) the ratio turns out to be <1. Hence, the probability of detecting the correct codeword is small compared to the probability of detecting a wrong codeword.

4. Expected Improvement

The following considerations are based on back-to-back transmission of NRZ-OOK data with 10 Gb/s net data rate. Optical bandwidth and electrical bandwidth for the optically preamplified receiver are set to 22 GHz and 7 GHz, respectively. For coded transmission, the data rate is set to 10.7 Gb/s and the electrical bandwidth is increased by a factor of 1.07 while the optical bandwidth is preserved. Fig. 2a depicts the BER vs. OSNR for the case of uncoded transmission obtained by Monte-Carlo simulations. For a priori estimation of the improvement due to the proposed method, coding gain due to regular encoding and decoding is determined by shifting this curve to the right by $10\cdot\log_{10}(1.07)$ dB=0.3 dB followed by computing the BER after correction by the following formula [5]:

$$BER_{out} = 1 - \sqrt[m]{1 - \sum_{r=t+1}^{n} \frac{r}{n} \cdot \binom{n}{r} \cdot \left[1 - \left(1 - BER_{in}\right)^{m}\right]^{r} \cdot \left(1 - BER_{in}\right)^{m(n-r)}}. \quad (2)$$

In (2) all contributions to the output BER in case of t+1 up to n symbol errors are summed up. In order to take into account the case that t+1 errors are corrected, the lower bound of the sum is set to t+2. The result is given in fig. 2a, too. For the regular decoding algorithm coding gain of 5.6 dB is obtained (BER=10^{-12}) while the proposed algorithm yields an improvement of 0.2 dB to result in coding gain of 5.8 dB.

5. Estimation of Effort

In the following, the derivation of the required number of decoding iterations (see figure 1) for a certain performance is approximated. We assume t+1=9 single bit errors distributed randomly within the whole block of 2040 bit. The probability of hitting one of those nine errors within the r^{th} iteration is given by

$$P(r) = \frac{9}{2041-r} \cdot \prod_{s=0}^{r-2} \frac{2031-s}{2040-s}. \tag{3}$$

The first elements are given by $P(1)$=9/2040, $P(2)$=9/2039·2031/2040 etc. Obviously, $P(r_0)>P(r_0+1)$, i.e. $P(r)$ decreases monotonically. The expectation $E\{P(r)\}\approx 204$, i.e. on the average the number of iterations required for successful decoding is equal to 10% of the total length of the block. The additional decoding overhead due to the proposed method can be estimated as follows: For achieving BER of 10^{-12} by the proposed method according to (2) an input BER of $2.54\cdot 10^{-4}$ is required. Using the conventional decoding algorithm correcting only 8 symbol errors, BER of $1.8\cdot 10^{-11}$ is achieved. Assuming that these errors are due to 9 symbol errors per block exclusively caused by single bit errors per symbol, decoder failure ratio of $1.8\cdot 10^{-11}/9=2\cdot 10^{-12}$ is obtained. This means that on the average after transmitting $1/(2\cdot 10^{-12})= 5\cdot 10^{11}$ bit 204 additional decoding procedures have to be carried out. Taking into account that for transmitting $5\cdot 10^{11}$ bit $5\cdot 10^{11}/2040=2.45\cdot 10^{8}$ regular decoding procedures are executed, an overhead of $204/2.45\cdot 10^{8}=8.33\cdot 10^{-5}$ % is obtained. This is negligible in a real system.

To provide enough time for the decoder to carry out the extended decoding algorithm, delay and appropriate buffering must be introduced. It is not necessary to carry out all 2040 iterations as (3) allows for showing that already after ≈580 iterations the probability for successful decoding is equal to 95% and for achieving a probability of 99% ≈820 iterations are required. The results obtained following this strategy are given in the next section.

6. Simulation Results

Based on Monte-Carlo simulations, Fig. 2b shows the BER as a function of OSNR in case of conventional decoding as well as for the proposed method. For the latter one, the curve is shown for the case that decoding is aborted after 1000 iterations. The curve approaches the theoretical limit and the decoder shows an improvement of ≈0.18 dB over the conventional method. As the optical filter is not adapted to the increased data rate, in contrast to fig. 2a coding gain of approx. 5.8 dB for the conventional method and approx. 6.0 dB for the proposed method are obtained.

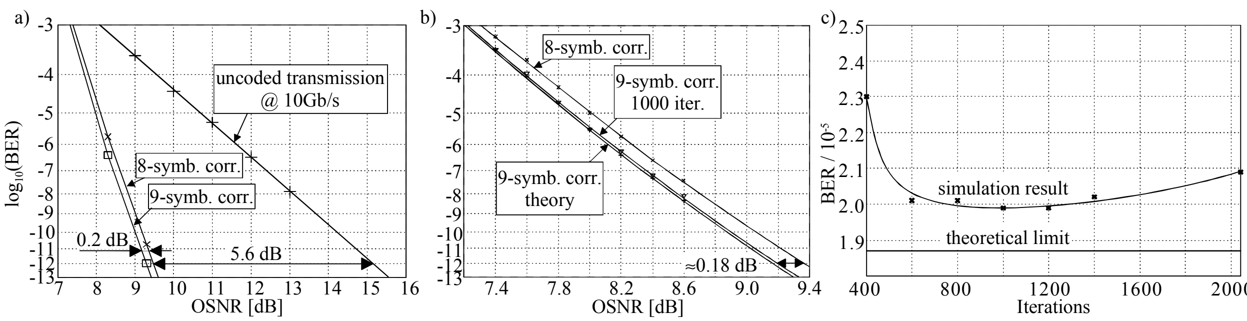

Figure 2: a) Expected coding gain for conventional and proposed method; b) simulation results for conventional method and for proposed method after 1000 iterations; c) BER as function of number of iterations for OSNR=7.8 dB.

The dependency of the BER on the number of iterations is given in fig. 2c for fixed OSNR of 7.8 dB: The optimum number of iterations is approximately equal to 1000: As for the proposed method decoding is performed beyond half of the minimum distance, occasionally a wrong codeword is detected. Obviously, it makes sense to abort the procedure after ≈50% of the block length as according to (3) the probability of hitting the first single error in the second half of the block is small compared to the probability of detecting a wrong codeword.

7. Conclusion

Without the need for using reliability information, simply by modifying the decoding procedure the error correction capability of standard FEC is increased by one symbol error. Numerical simulations for a standard back-to-back scenario show a coding gain of 6.0 dB (BER=10^{-12}).

References

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