Net Coding Gain of 10.2 dB using an irregular LDPC code with a Three-dimensional Analyser

Stefan Schöllmann, Oren Jean and Werner Rosenkranz
Chair for Communications, University of Kiel, Kaiserstr. 2, D-24143 Kiel, Germany
sts@tf.uni-kiel.de

Abstract: We present a three-dimensional decoding scheme for an irregular Low Densitiy Parity Check code (LDPC). With this setup, we achieved a Net Coding Gain of 10.2 dB and a significant improvement in the iterating decoding process.

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1. Introduction

Modern optical transmission systems need increasingly powerful forward error correction (FEC) schemes to ensure the maximum accuracy of data transmission with the minimum of optical power. During the last few years, a significant effort has been made to investigate different codes like concatenated codes [1], turbo codes [2] and Low Density Parity Check codes (LDPC) [3]. Recently, it was shown [4] that the error performance and the hardware complexity of FEC could be greatly improved by using irregular LDPC codes. In this paper we present a new three-dimensional decoding scheme and achieve a net coding gain (NCG) of 10.2 dB for an optical transmission system, which is limited by ASE noise.

The structure of the paper is as follows: In section 2 we describe the general simulation setup. Section 3 depicts in detail the structure of the three-dimensional analyser as the heart of our three dimensional decoding scheme. In section 4, the performance of the irregular LDPC code is presented in comparison to several other codes, and the performance advantage of the new decoding scheme is shown.

2. General setup

All simulation results are based on a single channel system shown in fig. 1. Thereby, our system is only influenced by ASE-noise of amplifiers. As we are primarily interested in the coding performance of the three-dimensional analyser, all other impairments (e.g. linear and nonlinear fiber influences) are neglected.

Fig: 1: General simulation setup for the evaluation of the three-dimenstional decoding scheme

The data generator generates random message blocks with a data rate of 10 Gb/s. In the next step, an LDPC encoder encodes these blocks. We used Vasic and Djordjevic H-matrix generation algorithm [3] to create a regular LDPC code (1369,1260) characterized by an overhead of only 8.7 %, a column weight of k=3 and a row weight of n=37, respectively. We removed any dependent rows in the generated H-matrix and get therefore an irregular LDPC code with the same redundancy of 8.7 %. These encoded data blocks are received at the three-dimensional analyser, which will be described in the following section. At the output of this three-dimensional analyser the likelihood of the received symbol to be a '1' or a '0' is calculated and is passed to the Soft Decision Iterative LDPC decoder. Finally the BER is determined by Monte-Carlo simulations.

3. Structure of the three-dimensional analyser

According to fig. 2a, the three dimensional analyser is composed of a splitter, three independent receivers (each consisting of an optical preamplifier, an optical filter, a pin diode detector and an electric filter), followed by A/D converters and one digital signal processing device.

The preamplifier in each receiver adds ASE noise to the optical signal. In our simulation, we approximated the noise after optical/electrical conversion and electrical filtering by a Gaussian distribution (see fig. 2b).

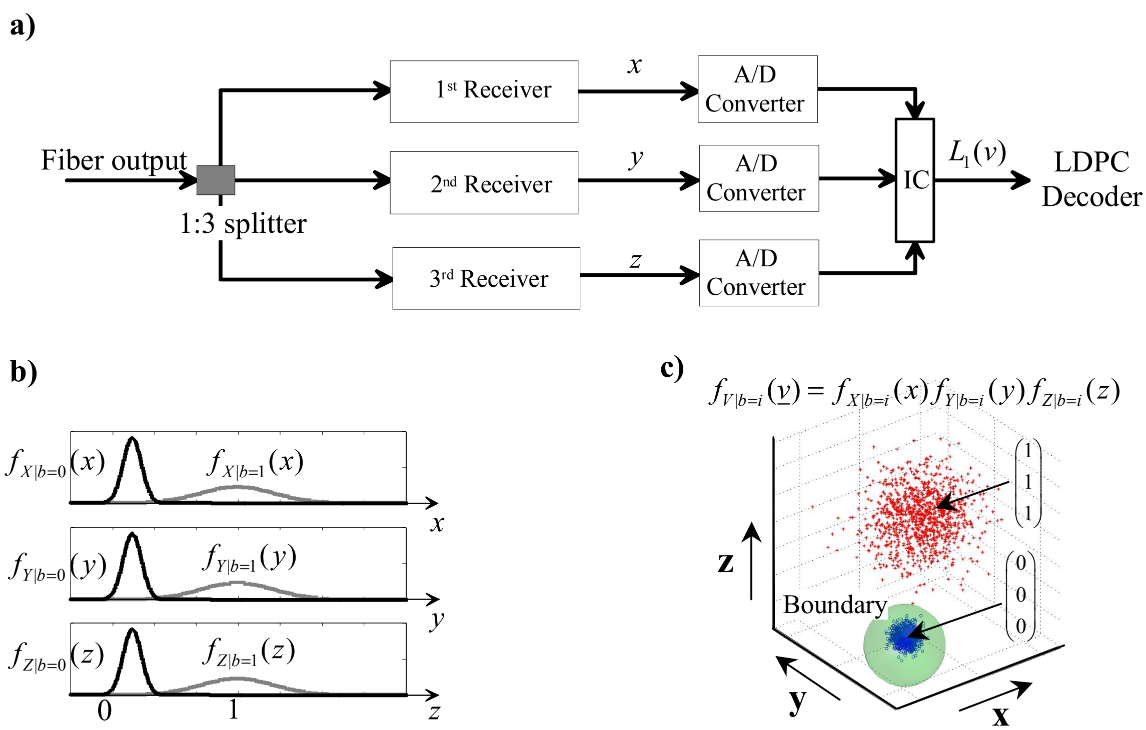

Fig. 2a: Structure of the three dimensional analyser with fiber output, demultiplexer, three receivers, A/D converters and an integrated circuit (IC); Fig. 2b: The conditional pdfs of the three receiver outputs; Fig. 2c: Scatter plot describing the separation in three-dimensional space

Furthermore we assumed that the noise in the three receivers is independent. This assumption was used due to the fact, that no significant correlation between the measurements of the receivers is expected (each one uses a different preamplifier). On this condition, we have three one-dimensional random variables *x*, *y* and *z* (see fig. 2a), from which three different conditional pdfs ($f_{X|b=i}(x), f_{Y|b=i}(y), f_{Z|b=i}(z)$ where $i \in \{0,1\}$) at the end of each receiver can be determined. The joint pdf of these variables can be calculated with equation 3.1:

$$f_{V|b=i}(\underline{v}) = f_{X|b=i}(x) f_{Y|b=i}(y) f_{Z|b=i}(z) \text{ where } \underline{V} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}, i \in \{0,1\}. \tag{3.1}$$

Due to this equation, it is now possible to calculate the likelihood $L_1(\underline{v})$ of the three-dimensional random vector to represent a '1' bit (The representation of a '0' bit can be determined by the complement of $L_1(\underline{v})$).

$$L_1(\underline{v}) = \frac{f_{V|b=1}(\underline{v})}{f_{V|b=1}(\underline{v}) + f_{V|b=0}(\underline{v})} \tag{3.2}$$

This likelihood value $L_1(\underline{v})$ is given finally as the output signal of the three-dimensional analyser to the iterative soft decision LDPC decoding algorithm [5].

In practice, sampling the output of the three receivers in the electrical domain generates the random variables *x*, *y* and *z*. This is done by three A/D converters [2]. Afterwards, the signal processing Integrated Circuit (IC) is responsible for the calculation of the Likelihood $L_1(\underline{v})$. Firstly, the corresponding one-dimensional pdf values to the random variables *x*, *y* and *z* are determined. In the next step, the joint pdf and the likelihood of the three-dimensional random vector are calculated based on the equations 3.1 and 3.2.

Fig. 2c shows the three-dimensional scatter plot of the sampled values ('1' and '0' bits) of the random vector $\underline{v}$ for an input power of –46 dBm into each of the three preamplifiers. The separation threshold, based on the classical Maximum Likelihood decision boundary, is represented in fig. 2c by the sphere around the '0'. Using three receivers for each symbol instead of only one gives a better separation possibility in the three-dimensional space compared to the one-dimensional space. Therefore a more accurate likelihood measure is created.

4. Results and Discussion

Fig. 3a and fig. 3b show the superior performance of the irregular LDPC code with our described three-dimensional analyser.

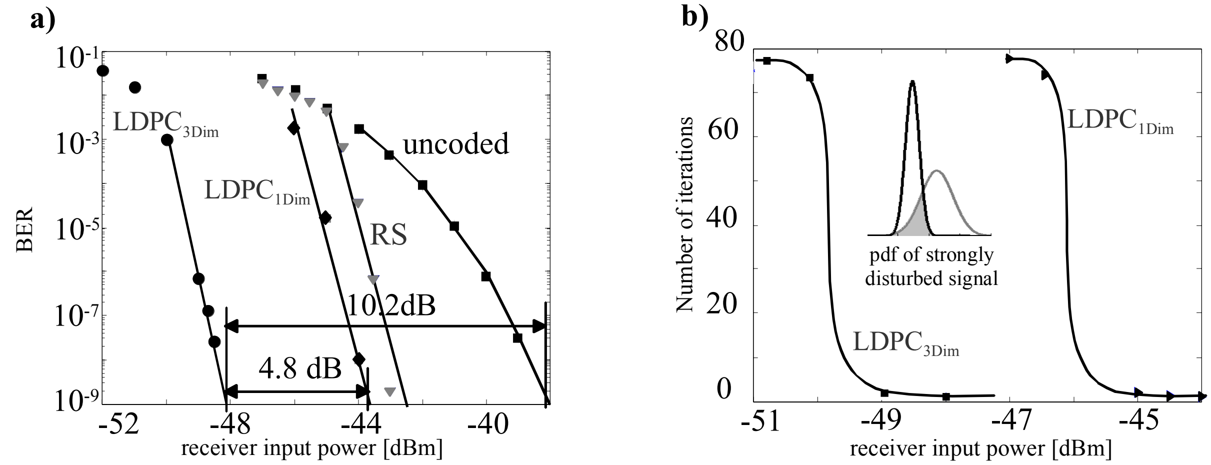

Fig. 3a: BER performance for different FEC codes: NCG of 10.2 dB and a gain of 4.8 dB compared to common irregular LDPC Coding scheme;
Fig. 3b: Number of iterations in the soft decoding process versus receiver input power for three-dimensional receiver and common receiver

Fig. 3a shows the BER versus the receiver input power for the investigated $LDPC_{3Dim}$ (1369,1260) FEC code compared to $LDPC_{1Dim}$ (1369,1260) and to the standard RS (255,239) as well as for the uncoded case. We achieve a total NCG of 10.2 dB using our three-dimensional analyser with an irregular LDPC code (only 8.7 % overhead) for a BER of 10^{-9}. Furthermore, the achieved gain from the common irregular LDPC Coding scheme is determined to be 4.8 dB (BER of 10^{-9}). To our knowledge, this is the best performance of an irregular LDPC code with such a low overhead.

Directly correlated to the achieved gain, we expect a significant advantage in the soft decision decoding algorithm which is mainly characterized by the number of iterations. In fig. 3b the number of iteration in the decoding process is shown versus the receiver input power. In our simulation the maximum number of iterations was arbitrarily set to 75. From fig. 3b it is apparent that the use of the three-dimensional analyser reduces drastically the required number of iterations for the same receiver input powers which confirms our expectations. Only for strongly disturbed signals ($BER \approx 10^{-2}$) as shown in the conditional pdf in fig.3b the maximum number of 75 iterations is necessary. Nevertheless, for a received input power of –48 dBm we never observed a decoded block that took more than one iteration. This is a very useful property for a real time implementation of several iterative decoding algorithms for LDPC codes.

5. Conclusion

We presented a new three-dimensional decoding scheme in combination with an irregular LDPC code (overhead of 8.7%) that outperforms previously reported one-dimensional FEC schemes. With this system upgrade we achieved a NCG of 10.2 dB or a gain of 4.8 dB compared to the common one-dimensional LDPC decoding scheme. Additionally, we showed that the usage of three-dimensional analyser as heart of our decoding scheme reduces drastically the number of iterations in the decoding process. This property could be exploited by real time implementation for several iterative decoding algorithms. Generally, it is possible to use the three-dimensional analyser in conjunction with an arbitrary FEC code.

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