

# Optimal Equalization of Distortions due to Group Delay Ripples of Chirped Fiber Bragg Gratings (CFBG)

Jan Mietzner, Sven Otte, Marc Bohn, Christoph Glingener and Werner Rosenkranz

**Abstract** We present a closed solution for an optimal equalizer which compensates for signal distortions caused by group delay ripples in chirped fiber Bragg gratings (CFBG) employed for dispersion compensation in optical communication systems. The theory is verified with the help of group delay measurement results of a dispersion compensating CFBG and the improvements achieved by the equalizer are presented on the basis of simulation results.

**Keywords** Equalization, Group delay ripples, Dispersion compensation, Chirped fiber Bragg grating (CFBG)

## 1. Introduction

The dramatically growing demand for data transport over optical networks forced higher single channel data rates and an increasing number of channels in wavelength-division multiplex (WDM) backbone systems. In this context, from an economic point of view, it is of particular interest to realize 40 Gb/s channels utilizing already installed fiber.

Usually standard-single-mode fibers (SSMF) are used in optical communication systems, especially in wide area networks (WAN). SSMF show chromatic dispersion, which corresponds to an approximately linear wavelength dependence of the group delay for light waves propagating along the fiber. As optical pulses used for data transmission contain a bunch of different wavelengths they therefore experience a broadening in the time domain. Especially in long fibers and in multi-span systems without 3R-regeneration (amplitude, clock and data recovery in the electrical domain) this can lead to a non-tolerable bit error rate (BER) due to intersymbol interference (ISI). Therefore in general dispersion and dispersion slope compensation is inevitable.

In order to eliminate the chromatic dispersion a cascade system of SSMF and dispersion compensating fiber (DCF) can be built. Alternatively, dispersion compensating chirped fiber Bragg gratings (CFBG) can be employed

in the cascade system. In both cases a total cancellation of the dispersion within the cascade system can be achieved, which complies with a constant group delay.

The application of CFBG features several advantages compared to the DCF as for example a smaller signal attenuation [1]. Moreover, CFBG are more compact and do not suffer from nonlinear effects. At the same time, however, they are characterized by a fabrication induced non-ideal phase response [2]. This leads to variations within their group delay (referred to as *group delay ripples*) which cause additional signal distortions [3] and limits the total optical transparent length of a link [4]. CFBGs are promising elements if it is possible to minimize the influence of the group delay ripples. In [5] the transmission range is increased by using CFBG which show small group delay ripples, employing a special fabrication process presented in [6].

In this paper an alternative approach is investigated. First of all a closed analytical theory is presented which describes the signal distortions caused by group delay ripples in a CFBG. The theoretical considerations lead to the derivation of an optimal equalizer which totally compensates for the signal distortions due to the non-ideal group delay characteristic of the CFBG. In the case of complete dispersion compensation a cascade system consisting of SSMF, CFBG and optimal equalizer will show a constant group delay and the additional signal distortions are cancelled. The optimal equalizer may be realized as integrated device by means of an optical transversal filter in planar lightwave circuit (PLC) technology.

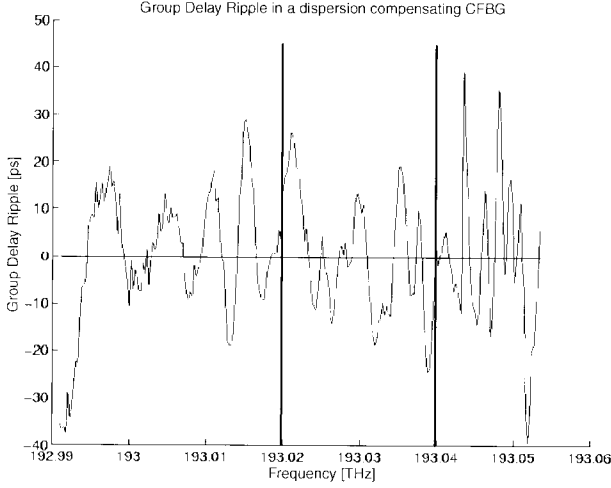
In Section 2 a model for the group delay ripples and the resulting signal distortions is developed on the basis of measurements within a dispersion compensating CFBG. In Section 3 the closed solution for the optimal equalizer is derived and in Section 4 simulation results are shown in order to verify the design of the equalizer. In Section 5 the results of this paper are summarized.

## 2. Model for the group delay ripples

In order to fabricate a dispersion compensating CFBG the phase mask employed to write the structure of the grating is linearly chirped. Due to this it is accomplished that the resulting group delay characteristic of the CFBG is a linear function of the wavelength with negative slope. Material properties and a non-ideal fabrication process result in the fact, that the linear characteristic is distorted which is referred to as *group delay ripples*. As an example

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**Fig. 1.** Group Delay Ripples according to a Measurement in a dispersion compensating CFBG.

Fig. 1 shows group delay ripples in a CFBG as a function of the absolute frequency as observed in a measurement. The vertical lines mark the boundaries of a single channel of 20 GHz bandwidth within the third optical window, which is considered in the derivations below.

When we assume that the chromatic dispersion brought in by the SSMF is completely compensated using a CFBG with an ideal phase response, then the group delay of the cascade system of SSMF and CFBG is constant. Therefore, when regarding a non-ideal CFBG the group delay ripples of the grating can be regarded as the group delay characteristic of the entire cascade system.

In order to develop a model for the occurring distortions due to the non-ideal group delay  $\tau_{\text{casc}}(f)$  of the cascade system a band in the third window is regarded. Within the regarded band it is possible to approximate the group delay by a sum of  $N < \infty$  Cosine functions:

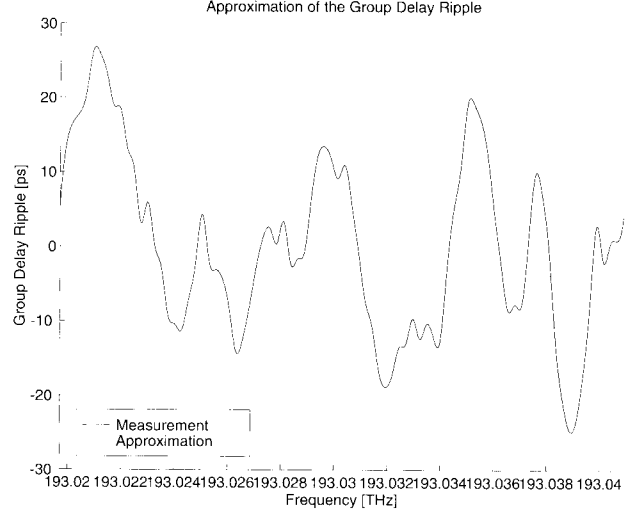
$$\tau_{\text{casc}}(f) = - \sum_{i=1}^N \Delta\tau_i \cos(2\pi f / f_{\text{ripple},i} + \varphi_i). \quad (1)$$

Each of the  $i \in \{1, \dots, N\}$  used Cosine functions shows an amplitude  $\Delta\tau_i$ , a period  $f_{\text{ripple},i}$  and an angular offset  $\varphi_i$ .

Integration of  $\tau_{\text{casc}}(f)$  leads to the phase response  $\phi_{\text{casc}}(f)$  and thus to the transfer function  $H_{\text{casc}}(f)$  of the cascade system. It is

$$\begin{aligned} H_{\text{casc}}(f) &= e^{-j\phi_{\text{casc}}(f)} \\ &= \prod_{i=1}^N e^{j\Delta\tau_i f_{\text{ripple},i} \sin(2\pi f / f_{\text{ripple},i} + \varphi_i)} \\ &= \prod_{i=1}^N H_i(f). \end{aligned} \quad (2)$$

Each of the  $N$  factors  $H_i(f)$  is associated with a partial impulse response



**Fig. 2.** Approximation of the Measured Group Delay Ripples using a sum of Cosine functions.

$$\begin{aligned} h_i(t) &= J_0(\Delta\tau_i f_{\text{ripple},i}) \delta_0(t) \\ &+ \sum_{k=1}^{\infty} J_k(\Delta\tau_i f_{\text{ripple},i}) \left[ \delta_0(t - k / f_{\text{ripple},i}) e^{jk\varphi_i} \right. \\ &\left. + (-1)^k \delta_0(t + k / f_{\text{ripple},i}) e^{-jk\varphi_i} \right], \end{aligned} \quad (3)$$

where  $J_k(x)$  denotes the Bessel function of the first kind of order  $k$  and  $\delta_0(t - x)$  is a Dirac impulse at  $t = x$  [3].

Finally, the impulse response of the cascade system can be obtained by a multiple convolution of the partial impulse responses  $h_i(t)$ :

$$h_{\text{casc}}(t) = h_1(t) * h_2(t) * \dots * h_N(t). \quad (4)$$

The measured group delay ripples of Fig. 1 have been approximated in the regarded band of the third window using a sum of  $N = 19$  Cosine functions according to (1). The values for the amplitudes  $\Delta\tau_i$ , the periods  $f_{\text{ripple},i}$  and the angular offsets  $\varphi_i$  have been obtained with the help of a Fast Fourier Transform (FFT) of the measured group delay ripples which is referred to as *ripple spectrum*. Fig. 2 shows the result of the approximation within the regarded band.

The values of  $\Delta\tau_i$ ,  $f_{\text{ripple},i}$  and  $\varphi_i$  employed in the approximation of the group delay have been used in order to calculate the impulse response  $h_{\text{casc}}(t)$  of the cascade system of SSMF and CFBG according to (4).

### 3. Optimal equalization

In this section we derive the optimal equalizer for cancellation of the group delay ripple distortions described in Section 2 assuming complete dispersion compensation.

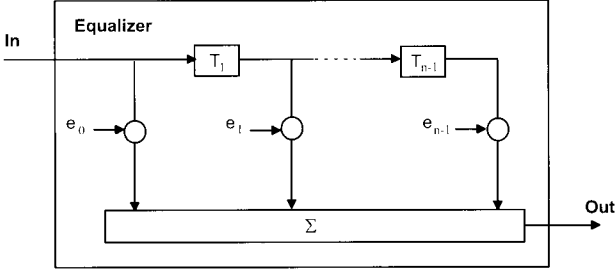


Fig. 3. Block Diagram of the Equalizer.

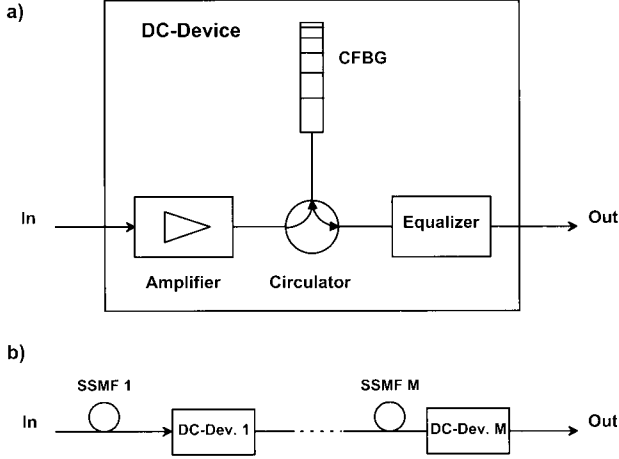


Fig. 4. Proposed Dispersion Compensating Device including a Group Delay Ripple Equalizer. b) Potential Application of the Device in a Multi-Span System.

The optimal equalizer  $E^{(\text{opt})}(f)$  can be identified as the inverse system of  $H_{\text{casc}}(f)$  according to (2), because in the case of optimal equalization the transfer function of the cascade system of SSMF, optical filter and equalizer has to be 1 (i.e. total cancellation of ISI, neglecting delays):

$$H_{\text{casc}}(f) E^{(\text{opt})}(f) \doteq 1. \quad (5)$$

According to (2) and (5) we obtain

$$\begin{aligned} E^{(\text{opt})}(f) &= \prod_{i=1}^N e^{-j\Delta\tau_i f_{\text{ripple},i}} \sin(2\pi f / f_{\text{ripple},i} + \varphi_i) \\ &= H_{\text{casc}}^*(f), \end{aligned} \quad (6)$$

where  $H_{\text{casc}}^*(f)$  denotes conjugation of the complex transfer function  $H_{\text{casc}}(f)$ .

Transfer of (6) to the time domain leads to the impulse response  $e^{(\text{opt})}(t)$  of the optimal equalizer (refer to (4)):

$$e^{(\text{opt})}(t) = h_{\text{casc}}^*(-t). \quad (7)$$

Therefore the optimal equalizer is a *Matched Filter* to the cascade system of SSMF and dispersion compensating CFBG. This result is as well valid in the case where

the chromatic dispersion of the SSMF is not completely compensated by the CFBG.

According to (4)  $h_{\text{casc}}(t)$  consists of an infinite number of coefficients. Therefore in practice it is necessary to choose a finite subset of coefficients  $\{e_0, \dots, e_{n-1}\}$  in order to design the impulse response of the equalizer ( $t_i$  is the delay associated with coefficient  $e_i$  with respect to  $e_0$ ):

$$e(t) = \sum_{i=0}^{n-1} e_i \delta_0(t - t_i). \quad (8)$$

The structure of the equalizer according to (8) complies with an FIR filter (*finite impulse response*). It is illustrated as a block diagram in Fig. 3, where  $T_i$  is the delay between the taps for the coefficients  $e_{i-1}$  and  $e_i$ .

The FIR structure of the equalizer can in principal be realized in form of a PLC, where it is possible to generate filter coefficients  $e_i$  showing arbitrary complex values [7].

Fig. 4 shows our proposal for a dispersion compensating device employing a CFBG and an appropriate equalizer, including an optical amplifier for loss compensation. Moreover, a potential application of the device in a multi-span system is shown leading to an increase of the total optical transparent length of a link (refer to Section 4).

## 4. Simulation results

In simulations we only took into account 4 coefficients of the optimal equalizer located at each side of the main impulse of  $e^{(\text{opt})}(t)$  which leads to an equalizer with  $n = 9$  coefficients (refer to Fig. 3,  $T_i$  was in the order of  $0.5/f_{\text{bit}}$ , where the bit rate  $f_{\text{bit}}$  was 10 Gbit/s). However, the simulations show that this suboptimal equalizer already yields good results.

Figs. 5 and 6 show the impulse response of the cascade system of SSMF and CFBG before and after equaliza-

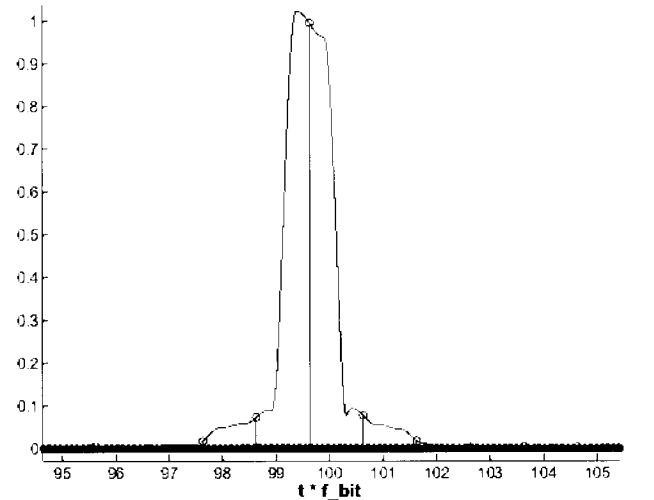
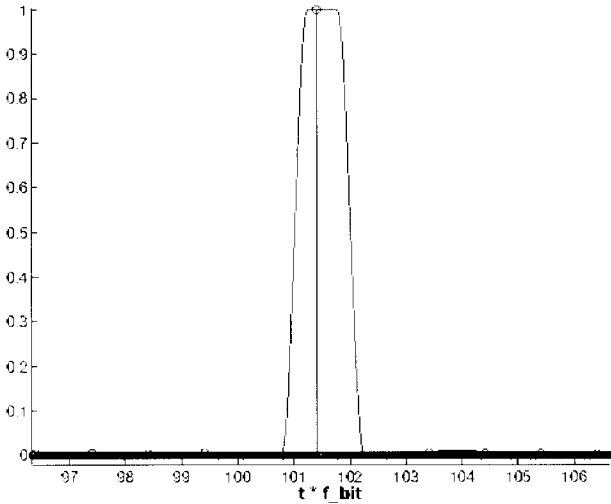


Fig. 5. Impulse Response of the Cascade System (SSMF + CFBG), no Equalization.



**Fig. 6.** Impulse Response of the Cascade System (SSMF + CFBG + Equalizer), 9 Coefficients used for the Equalization.

tion (1 span). For the cascade system we assumed a pulse shape of limited bandwidth showing a Cosine edge in the time domain.

Dispersion compensating CFBG realistically show a dispersion in the order of  $-1400$  ps/nm. If after one span a total compensation for the occurring dispersion is desired, a span is therefore correlated with a fiber length

in the order of 80 km, as SSMF shows a dispersion of 17 ps/(nm km) in the third window.

In order to quantify the ISI brought in by each impulse response, the regarded impulse response is sampled according to the bit rate  $f_{bit}$  and the optimal sample time (in the sense of minimal ISI) is chosen, as shown in Figs. 5 and 6. The optimal samples are normalized to a maximum value of 1 yielding a vector  $\mathbf{s}$ , which is used in order to define an *ISI-penalty (ISIP)* for the regarded impulse response. The *ISIP*-values in dB are defined on the basis of the Euclidean norm  $\|\cdot\|_2$  of  $\mathbf{s}$ :

$$ISIP = 10 \log_{10}(\|\mathbf{s}\|_2^2) \text{ dB.} \quad (9)$$

Without equalization an *ISIP*-value of

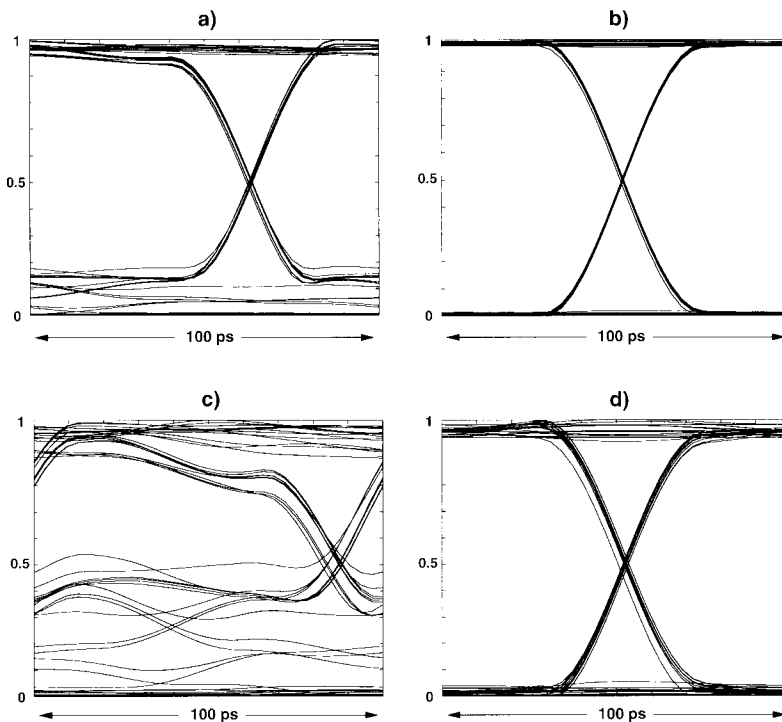
$$ISIP_0 = 0.0204 \text{ dB} \quad (10)$$

was calculated, whereas the equalized impulse response lead to an *ISIP*-value of

$$ISIP_{cq} = 8.5243 \cdot 10^{-4} \text{ dB.} \quad (11)$$

This marks a significant ISI compensation accomplished by the equalization.

In Fig. 7 the improvement achieved by the equalization process is illustrated with the help of eye diagrams. Fig. 7 a) and b) show eye diagrams after a single span, whereas in Fig. 7 c) and d)  $M = 3$  spans are regarded (refer to Fig. 4).



**Fig. 7.** Cascade System (SSMF + CFBG (+ Equalizer)) a) Eye Diagram after 1 Span (no Equalization) b) Eye Diagram after 1 Span, Equalization with 9 Coefficients c) Eye Diagram after 3 Spans (no Equalization) d) Eye Diagram after 3 Spans, Equalization with 9 Coefficients after each Span.

In Fig. 7 a) and c) no equalizer is employed. Figs. 7 b) and d) show the eye diagrams obtained after an equalization using  $n = 9$  coefficients, according to Eqs. (6)–(8), where in the latter case the equalization is done after each span. In the case of Fig. 7 a) the eye diagram already shows a significant degree of degradation, which can lead to a non-tolerable bit error rate, if additional noise is considered. In the case of Fig. 7 c) it is clear that either 3R regeneration or equalization of the received signal is inevitable, even if there is very little additional noise.

## 5. Conclusion

In Section 2 we presented a model for signal distortions, which appear in optical communication systems when CFBG are used in order to compensate for the chromatic dispersion brought in by an SSMF. The model was derived from group delay measurements within a CFBG.

On the basis of the signal distortion model we presented a closed solution for an optimal equalizer in Section 3. A practical realization of the equalizer in form of a PLC was proposed, as the equalizer turned out to show an FIR structure.

The simulation results of Section 4 showed that the ISI within the cascade system of SSMF and CFBG can significantly be reduced although only a suboptimal equalizer with a rather small number of coefficients was used in the simulations. This has been shown with eye diagrams as well as with values for the corresponding ISI-penalties. It has been shown, that the reduction of ISI is particularly useful in multi-span systems and embodies an alternative approach to 3R regeneration after each span.

## Appendix

### Abbreviations

3R - Signal regeneration in the electrical domain  
 BER - Bit error rate  
 CFBG - Chirped fiber Bragg grating  
 DCF - Dispersion compensating fiber  
 FIR - Finite impulse response  
 FFT - Fast Fourier transform  
 ISI - Intersymbol interference  
 ISIP - ISI penalty  
 PLC - Planar light wave circuit  
 SSMF - Standard-single-mode fiber  
 WAN - Wide area network  
 WDM - Wavelength-division multiplex

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