A DECISION FEEDBACK EQUALIZER FOR DISPERSION COMPENSATION IN HIGH SPEED OPTICAL TRANSMISSION SYSTEMS

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Abstract: We investigate a class of decision feedback equalizers, which are suitable for the equalization of dispersion effects in optical fiber channels. Since the widely used standard single mode fiber can be described by a simple linear transfer function for the optical signal, the overall system, including electrical/optical conversion and vice versa is nonlinear. We derive equalizer structures that account approximately for the nonlinear nature. These structures are investigated by simulation. An improvement of approx. 10 dB in the BER is achieved with a simple 1^{st} order equalizer that may be implemented at high speed systems at 10 Gb/s and beyond.

I. Introduction

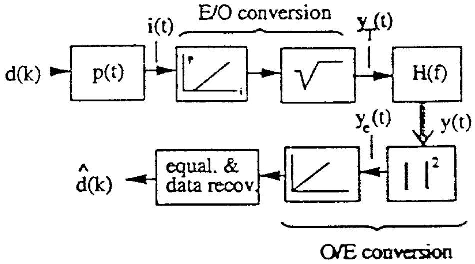
Equalization techniques in the electrical part of the optical receiver, i.e. after the optical/electrical conversion, have become of interest [1,3,4], because they are suitable to overcome transmission impairments such as chromatic and polarization dispersion, nonlinear fiber performance or WDM intermodulation effects. These methods are attractive, since they may be integrated via receiver chip design and thus, if once implemented, may become a much cheaper solution, compared to optical compensation methods, e.g. the use of dispersion compensating fiber [6]. Equalization of the optical channel after the optical/electrical conversion is difficult however, because of the nonlinear character of the link. We investigate electrical decision feedback equalizers (DFE) which are used for chromatic dispersion compensation in high speed fiber links. The concepts are then simplified, so that a hardware solution is possible, even at very high speed links. It is shown, that an improvement of approx. 10 dB can be achieved for a simple first order DFE.

The outline of this paper is as follows: In section II the simulation model is discussed in order to motivate why a decision feedback equalizer with a nonlinear feedback system can increase the performance of the equalization. The dispersion compensation methods and the determination of the equalizer coefficients are outlined in sections III and IV. Numerical results in terms of the bit-error-probability and the eye opening are shown in section V. We close with some concluding notes in section VI.

II. Simulation Model

A PRBS (pseudo random binary sequence) signal *d(k)* is generated and filtered by a linear filter with impulse response *p(t)* which has a sine square rise and fall characteristic in order to account for the limited bandwidth of the transceiver. This nonreturn to zero (NRZ) input data stream is used as a modulating signal which is assumed to be chirp-free. The complex envelope at the fiber input is $y_T(t)$ which is proportional to the square root of the optical laser output power. The dominant source of distortion is the chromatic dispersion in the fiber. The frequency response of the fiber is given by $H(f) = \exp(-jbf^2)$, $b = \pi D(\lambda_T)\,\lambda_T^2 / c\,L$, with λ_T the transmission wavelength and the dispersion coefficient $D(\lambda_T)$ for standard single mode fibers (SSMF), L the fiber length and c the speed of light. *H(f)* is the Fourier-transform of the impulse response *h(t)* of the fiber and thus the fiber output signal is $y(t)=h(t)*y_T(t)$. The PIN diode at the receiver converts the electrical field into an electrical signal proportional to the signal power $y_e(t)=|y(t)|^2$. Finally the combined equalization and data recovery generates the estimated data output $\hat{d}(k)$.

Figure 1: Simulation model



The overall system is thus nonlinear due to the square root and the magnitude squared operation even though the fiber itself is a linear filter. Figure 1 shows the simulation model.

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III. Dispersion Compensation

We estimate a linear impulse response $g(t)$ of the system: laser, modulator, and fiber. Thus we have neglected the nonlinear influence of the square root law of the transmitter. The system is modeled in the equivalent baseband hence the optical signal is complex $y=y_r+jy_i$. The optical receiver input signal after sampling is in good approximation given by (m=2, by inspection of the impulse response) :

$$y(k) \approx d(k)g(0)+d(k-1)g(1)+d(k-2)g(2) \quad (1)$$

With $g(i)=g_r(i)+jg_i(i),\ i=0,1,2$ the real and imaginary part of the sampling values of g. The converted electrical signal is then

$$y_e(k)=|y(k)|^2 \approx Ad(k)+Bd(k-1)+Cd(k-2)+Dd(k)d(k-1) + Ed(k)d(k-2)+Fd(k-1)d(k-2) \quad (2)$$

where A,B,C,D,E,F are constants which only depend on the sampling values of the impulse response $g(t)$:

$$A=|g(0)|^2,\ B=|g(1)|^2,\ C=|g(2)|^2$$
$$D=2Re\{g(1)g^*(0)\},\ E=2Re\{g(2)g^*(0)\},$$
$$F=2Re\{g(1)g^*(2)\} \quad (3)$$

(* denotes conjugate complex). Solving equation (2) for the actual bit $d(k)$ which has to be decided yields

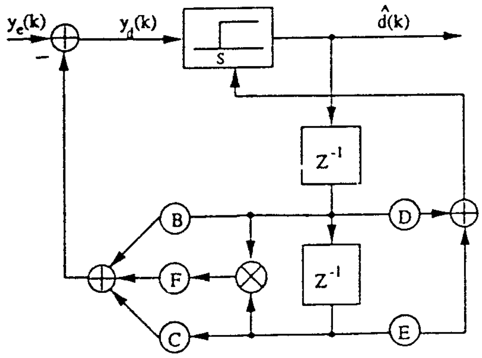
$$y_d(k)=d(k)[A+Dd(k-1)+Ed(k-2)] \approx y_e(k)-Bd(k-1)-Cd(k-2)-Fd(k-1)d(k-2) \quad (4)$$

This expression describes a modified decision feedback equalizer, which we call "squared DFE". The decision of the actual bit depends on the received signal $y_e(k)$ and the already detected bits $d(k-1)$, $d(k-2)$. On the right hand side of Eq. (4) the linear and nonlinear part of the intersymbol interference is subtracted from the received electrical signal $y_e(k)$. Since the threshold S in the decision device of the receiver is constant for an ordinary decision feedback equalizer, the left hand side of Eq. (4) describes a data dependent decision threshold. The resulting signal after processing equation (4) is the input $y_d(k)$ of the hard limiter respectively the decision device. The output of the hard limiter is then the estimated data sequence $\hat{d}(k)$. Fig. 3 shows the squared equalizer. The optimal decision threshold in the case $d(k-1)=d(k-2)=0$ is $S=A/2$. In the other cases the decision threshold as a function of the already detected data is according to Eq. (4)

$$S(d(k-1),d(k-2))=\begin{cases}\frac{A+D}{2}, & d(k-1)=1, d(k-2)=0\\ \frac{A+E}{2}, & d(k-1)=0, d(k-2)=1\\ \frac{A+D+E}{2}, & d(k-1)=d(k-2)=1\end{cases} \quad (5)$$

The performance of the squared DFE will be evaluated and compared with simplified structures in section V. These simplified structures are derived from (4) without taking into account the nonlinear absolute square operation by omitting coefficients F,D, and E. This results directly in a 2[nd] order DFE. A very simple 1[st] order DFE with only one coefficient results if additionally $d(k-2)$ can be neglected. The remaining transversal filter is just a one tap delay line with the tap spacing T_b.

Figure 2: Model of the squared equalizer



IV. Coefficient Determination

In general the system impulse response $g(t)$ is unknown or may change considering time variant systems for example variable fiber length or variable number of channels in a WDM (wavelength division multiplex) network. Therefore an adaptive determination is an important task.

It has to be taken into consideration that the received signal $y_e(k)$ does not in general obey exactly the system model equation (2). This is in the simulation according to Fig. 1 mainly due to the modeling approximation (1) and, if an experimental setup is considered, additionally due to implementation impairments e.g. noise. Since equation (2) is a linear combination of the coefficients (2) can be written as

$$y_e(k)=K^T d(k)+r(k) \quad (6)$$

where $K=[A\ B\ C\ D\ E\ F]^T$ is the coefficient vector and $d(k)=[d(k)\ d(k-1)\ d(k-2)\ d(k)d(k-1)\ d(k)d(k-2)\ d(k-1)d(k-2)]^T$ is the vector containing the data. We have added the model error $r(k)$. Minimizing the mean square error (MSE)

$$F^1_{MSE}=\sum_{k=0}^{\infty}\left(y_e(k)-K^T d(k)\right)^2 \quad (7)$$

adaptively with the help of the least mean square (LMS) algorithm [5] leads to a coefficient vector K. Our simulation shows that this coefficient vector K is not applicable for the equalization according to Eq. (4). This is because the MSE-error in Eq. (7) does not maximize the eye opening or the BER respectively. Transforming Eq. (6) gives :

$$d_r(k)=d(k)+\frac{r(k)}{A+Dd(k-1)+Ed(k-2)}=\frac{y_e(k)-Bd(k-1)-Cd(k-2)-Fd(k-1)d(k-2)}{A+Dd(k-1)+Ed(k-2)} \quad (8)$$

What we really want to minimize is the error

$$F_{MSE}^2 = \sum_{k=0}^{\infty} \left(d(k) - d_r(k)\right)^2 \tag{9}$$

However Eq. (8) is no longer a linear combination of the coefficients and thus the linear adaptive algorithms are not applicable. Therefore we propose the following 2-step method. In the first step the error

$$F_{MSE}^3 = \sum_{k=0}^{\infty} \left(d(k) - \left(y_e(k) - [B\,C\,F] \begin{bmatrix} d(k-1) \\ d(k-2) \\ d(k-1)d(k-2) \end{bmatrix} \right) \right)^2 \tag{10}$$

is minimized adaptively resulting in B, C and F. In a second step the constants A, D and E are determined as indicated in Eq. (5). Instead of calculating the decision threshold S with the help of A, D and E, we will determine optimal decision thresholds for the 4 cases $[d(k-1), d(k-2)] = [0, 0], [0, 1], [1, 0]$ and $[1, 1]$. Let these thresholds be S_{00}, S_{01}, S_{10} and S_{11}. This results in four linear equations to be solved for the constants A, D and E:

$$S = \begin{bmatrix} S_{00} \\ S_{01} \\ S_{10} \\ S_{11} \end{bmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} A \\ D \\ E \end{pmatrix} \doteq T\tilde{K} \tag{11}$$

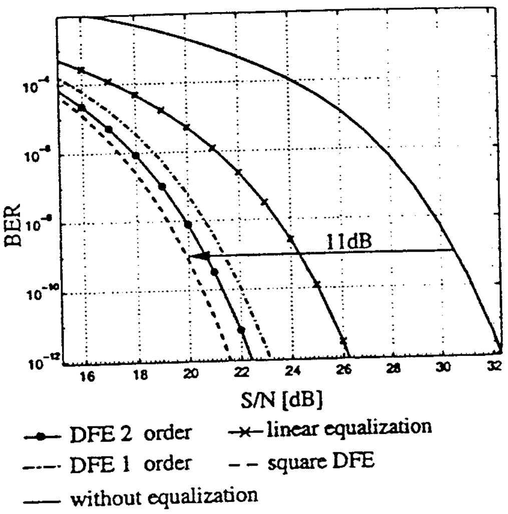
The optimal coefficient vector $\tilde{K} = (A\,D\,E)^T$ is then

$$\tilde{K} = \left(T^T T\right)^{-1} T^T S \tag{14}$$

V. Results

We will compare the proposed equalization approaches (the DFE 1st and 2nd order and the squared DFE) in terms of Bit-Error-Ratio (BER), eye opening, and repeater distance. We assume standard single mode dispersive fiber with $D(\lambda_T)$=17 ps/nm/km at 10 Gb/s at λ_T=1550nm. In Fig. 3 simulation results of the BER after a fiberlength of 130km are shown applying the different approaches. We assume additive white Gaussian noise. The BER is calculated for each sample at the input of the decision device by solving the integral of the probability density function (PDF) of the noise. This results in an analytic expression which mainly contains the error function complement. Hence this method is called the quasi-analytic (QA) method since it is a mix of simulation and analytic calculation. The total BER is derived by the mean of the sample BERs. As expected the squared DFE, which takes explicitly the magnitude square operation into account, is superior to the other approaches. For comparison the BER curve resulting after linear equalization is entered in Fig 3. The linear equalization filter with the impulse response $e(t)$ is designed so that with

Figure 3: BER, 130km standard single mode fiber, 10Gb/s



$$y_e(t) * e(t) = \sum_{i=0}^{\infty} d(i) p(t - iT_B - t_0) + \varepsilon(t) \tag{13}$$

the MSE

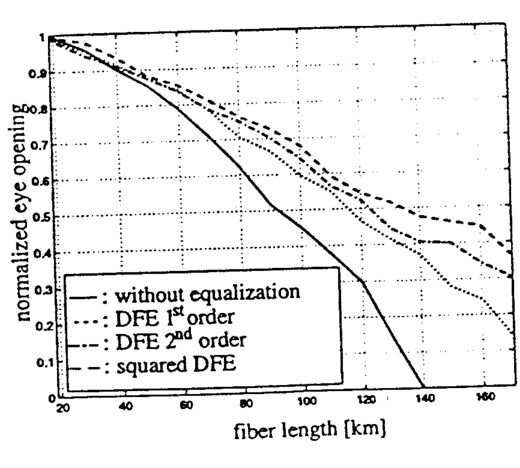
$$\lim_{T \to \infty} \frac{1}{T} \int_{t=0}^{T} \varepsilon^2(t)\, dt \tag{14}$$

is minimized. This linear filter is the optimal linear equalization filter in the MSE sense for the nonlinear system. It is obvious that the improvement applying the feedback approaches are much more significant.

The resulting BER after DF 2nd order equalization and even after DF 1st order equalization is close to the squared DFE. As indicated in Fig. 3 the BER improvement is more then 11dB at a BER of 10^{-9}.

Error propagation due to the feedback of erroneous bits was investigated [2] by Monte Carlo (MC) simulation. It has been sown, that for a BER<10^{-5} the influence is negligible.

Figure 4 : eye opening vs fiber length



Analytically for an ideal linear real valued channel with one post cursor echo $g(1)$ one can show that the BER without equalization is

$$BER_0 = \frac{1}{4}\left[erfc\left(\frac{g(0)-g(1)}{2\sqrt{2}\sigma}\right) + erfc\left(\frac{g(0)+g(1)}{2\sqrt{2}\sigma}\right)\right] \quad (15)$$

and for a 1[st] order DFE

$$BER = \frac{\frac{1}{2} erfc\left(\frac{g(0)}{2\sqrt{2}\sigma}\right)}{1-\frac{1}{4} erfc\left(\frac{g(0)-2g(1)}{2\sqrt{2}\sigma}\right)} \quad (16)$$

is a worst case approximation of the BER. In a practical high speed optical transmission system the denominator of equation (16), which describes the error propagation, becomes very small.

In Figure 4 the evolution of the normalized eye opening with the fiber length applying DFE 1[st] and 2[nd] order as well as the squared equalizer is shown.

Without equalization the remaining eye opening after 120 km is less than 30 %, whereas after equalization with the simple DFE 1[st] order the eye opening is 48% and reaches 56% applying the more complicated squared equalizer. Therefore the repeaterless distance could be increased from 90km to more than 120 km, if we accept an eye opening of 50 %. Note that the equalization of the 2[nd] post cursor echo $g(2)$ becomes more important with increasing fiber length due to the increasing dispersion.

Fig. 5 shows an eye diagram after a fiber length of 130km. The useful eye-contour is marked by a '+'. A secure data decision is impossible in practice due to the implementation impacts e.g. noise and sampling time jitter. Applying the DFE 1[st] order the improved eye diagram is shown in Fig. 6 as an example. The eye opening is substantially wider in both the horizontal and the vertical axis. Thus the DFE receiver should also result in an improved jitter performance.

VI. Conclusion

In this paper we have proposed various types of decision feedback equalizers for dispersive optical channels. The very simple DFE of 1[st] order is the best compromise between performance and implementation complexity. In a future step additional channel impairments as polarization mode dispersion and nonlinear fiber phenomena in the high power region will be considered

Figure: 5: eye diagram without equalization after L=130km of standard single mode fiber 10Gb/s

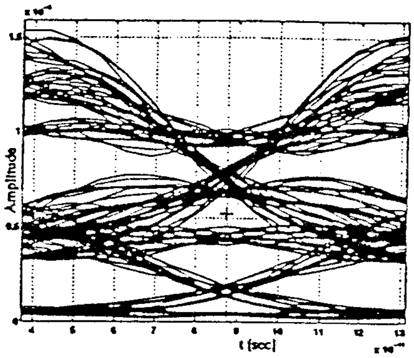
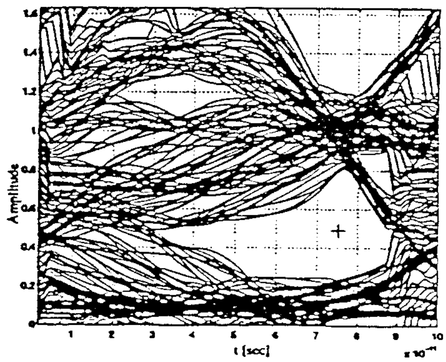


Figure 6: eye diagram DFE 1[st] order after L=130km of standard single mode fiber 10Gb/s



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